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32 — ORDINATE SCHEME OF ANALYSIS FOR CASCADE WAVES FROM TIME SERIES DATA*

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ABSTRACT

Diurnal, semidiurnal and quarterdiurnal variations (Cascade waves) are noticed not only in biological systems, but also in environmental conditions like the barometric pressure and tide level changes, etc. Quantitative studies of such variations require special type of harmonic analysis upto the fourth harmonic, but excluding the third harmonic. A 32-ordinate scheme is formulated for quick results of such analysis. The scheme is applied in rhythm studies of cell activity in DNA synthesis, as an example. The scheme provides an accurate and lucid means for analysis of cascade waves from time series data.

INTRODUCTION

BASED on harmonic analysis (Fourier Series), Runge (1902, 1905) formulated simple analytical schemes for obtaining serially the first 3, 6 or 12 waves, without excluding any wave in each set. But, we come across cascade variations, especially in case of variables controlled by sun. In such cases, we need to determine the fourth harmonic (the wave period one-fourth of that of the primary wave) together with the second harmonic (the wave period being one-half of that of the primary wave) and the primary wave *i.e.* upto the fourth harmonic except the third one. Such cascade system of waves was analysed from time series data by a selection of 8-ordinates equidistantly placed along the time axis, starting from t = 0 (Murty, 1978). The 8-ordinate scheme is defective in the sense that the sine factor of the last wave (with wave period one-fourth of the primary period *i.e.* with frequency 4) is absent in it. This is the case with any scheme (including the Runge schemes) with respect to the wave number which coincides with a half of the number of ordinates at choice, as it corresponds to the integral multiple of the angle π the sine value of which is therefore zero. The method was improved later by selecting 16-ordinates (Murty, 1987). The more the number of ordinates involved in the analysis, the better would be the accuracy of the results. Therefore, 32-ordinate scheme is evolved to transform the variations of a parameter over a time of primary period into cascade waves of four periods. An example is worked out by making use of the scheme in rhythm studies of cell activity in DNA synthesis.

32-ORDINATE SCHEME

In the scheme, the primary period T is divided into 32 equal parts and the corresponding 32 ordinates starting from the ordinate when time t is zero, are taken into consideration for analysis. Treating the complete cycle of the primary wave as 2π radians, the sine and cosine terms of all the angles involved in this division each reduce in magnitude to one of the values of 0, 0.195, 0.38, 0.56, 0.71, 0.83, 0.92, 0.98 or 1.

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Let the ordinates be $Y_0, Y_1, Y_2, \ldots, Y_{30}$ and Y_{31} where the suffix refers to the sequence of ordinates. Arrange the ordinates as

As indicated in the above Table the coefficients a_0, a_1, a_2, a_4, b_1, b_2 and b_4 are determined to frame the cascade waves upto the fourth order period.

$$\begin{array}{l} Y_1\ Y_2\ Y_3\ Y_4\ Y_5\ Y_6\ Y_7\ Y_8\ Y_9\ Y_{10}\ Y_{11}\ Y_{12}\ Y_{13}\ Y_{14}\ Y_{15}\ Y_{16} \\ Y_{31}\ Y_{30}\ Y_{29}\ Y_{28}\ Y_{27}\ Y_{26}\ Y_{25}\ Y_{24}\ Y_{23}\ Y_{22}\ Y_{21}\ Y_{20}\ Y_{19}\ Y_{18}\ Y_{17} \end{array}$$

p Sum $P_0\ P_1\ P_2\ P_3\ P_4\ P_5\ P_6\ P_7\ P_8\ P_9\ P_{10}\ P_{11}\ P_{12}\ P_{13}\ P_{14}\ P_{15}\ P_{16}$

q Diff. $q_1\ q_2\ q_3\ q_4\ q_5\ q_6\ q_7\ q_8\ q_9\ q_{10}\ q_{11}\ q_{12}\ q_{13}\ q_{14}\ q_{15}$

Rearrange p and q series as

$$\begin{array}{l} p_0\ p_1\ p_2\ p_3\ p_4\ p_5\ p_6\ p_7\ p_8 \\ p_{16}\ p_{15}\ p_{14}\ p_{13}\ p_{12}\ p_{11}\ p_{10}\ p_9 \end{array} \qquad \begin{array}{l} q_7\ q_2\ q_3\ q_4\ q_5\ q_6\ q_7\ q_8 \\ q_{15}\ q_{14}\ q_{13}\ q_{12}\ q_{11}\ q_{10}\ q_9 \end{array}$$

r $r_0\ r_1\ r_2\ r_3\ r_4\ r_5\ r_6\ r_7\ r_8$ Sum t $t_1\ t_2\ t_3\ t_4\ t_5\ t_6\ t_7\ t_8$

s $s_0\ s_1\ s_2\ s_3\ s_4\ s_5\ s_6\ s_7$ Diff. u $u_1\ u_2\ u_3\ u_4\ u_5\ u_6\ u_7$

Rearrange r and u series as

$$\begin{array}{l} r_0\ r_1\ r_2\ r_3\ r_4 \\ r_8\ r_7\ r_6\ r_5 \end{array} \qquad \begin{array}{l} u_1\ u_2\ u_3\ u_4 \\ u_7\ u_6\ u_5 \end{array}$$

v $v_0\ v_1\ v_2\ v_3\ v_4$ Sum k $k_1\ k_2\ k_3\ k_4$

w $w_0\ w_1\ w_2\ w_3$ Diff l $l_1\ l_2\ l_3$

and v series as

$$v_0\ v_1\ v_2\ v_4\ v_3$$

m Sum $m_0\ m_1\ m_2$

n Diff. $n_0\ n_1$

The results are finally tabulated as

Multiplier							
0.195		s_7			t_1		
0.38		s_6	w_3		t_2	k_1	
0.56		s_5			t_3		
0.71		s_4	w_2	n_1	t_4	k_2	$l_1 + l_3$
0.83		s_3			t_5		
0.92		s_2	w_1		t_6	k_3	
0.98		s_1			t_7		
1	$m_0 + m_1 + m_2$	s_0	w_0	n_0	t_8	k_4	l_2
Sum of : column :	$32a_0$	$16a_1$	$16a_2$	$16a_4$	$16b_1$	$16b_2$	$16b_4$

With the aid of the coefficients, the function of variable parameter y at any time t is given by

$$y = a_0 + a_1 \cos \frac{(a\pi\, t)}{T} + a_2 \cos 2\,\frac{(2\pi\, t)}{T}$$

$$+ 2_4 \cos 4\frac{(2\,\pi\, t)}{T} + b_1 \sin \frac{(2\,\pi\, t)}{T}$$

$$+ b_2 \sin 2\frac{(2\,\pi\, t)}{T} + b_4 \sin 4\frac{(2\,\pi\, t)}{T}$$

$a_1 \cos \frac{(2\,\pi\, t)}{T} + b_1 \sin \frac{(2\,\pi\, t)}{T}$ is the primary wave whose period is T, $a_2 \cos 2\frac{(2\,\pi\, t)}{T} + b_2 \sin 2\frac{(2\,\pi\, t)}{T}$ is the secondary wave (second harmonic) whose period is $\frac{T}{2}$ and $a_4 \cos 4\frac{(2\,\pi\, t)}{T} + b_4 \sin 4\frac{(2\,\pi\, t)}{T}$ is the quarter period wave (fourth harmonic). The four cascade waves ride over the steady value (a_0) of the parameter with appropriate phase angles. The amplitudes of the respective waves are given by $A_1 = \sqrt{a_1^2 + b_1^2}$ $A_2 = \sqrt{a_2^2 + b_2^2}$, and $A_4 = \sqrt{a_2^2 + b_4^2}$

Worked out example

The 32-ordinate scheme is applied to circadian rhythms in the number of living cells involved in DNA synthesis, as observed by Nieto *et al.* (1987) in their experiment in cell proliferation activity of goldfish intestine. The number of such cell counts per fold section were pertaining to 24 hour at interval of 1 h (Fig. 1). The cycle is complete by 24 h *i.e.* the cell count for 0 h and 24 h is the same. If not so, the mean value for both the hours is to be considered to represent 0 h or 24 h value. The observed values are plotted on a graph sheet and the points are jointed in sequence by straight lines in order to make it convenient to choose the 32 ordinates at appropriate intervals of time. In the present case the 24 h divided by 32 gives 0.75 h. Therefore the required ordinates are spaced at intervals of 0.75 h and we have to start counting the ordinates from that corresponding to starting time (t = 0 h). The required ordinates obtained from the graph are as follows :

Y_0	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8
10.8	12.9	13.4	11.7	7.1	10.5	11.1	9.8	7.3

Y_9	Y_{10}	Y_{11}	Y_{12}	Y_{13}	Y_{14}	Y_{15}	Y_{16}	Y_{17}
4.0	1.8	1.4	2.4	1.3	0.9	1.5	3.2	2.7

Y_{18}	Y_{19}	Y_{20}	Y_{21}	Y_{22}	Y_{23}	Y_{24}	Y_{25}
4.0	5.5	5.8	6.8	6.9	7.2	9.6	13.0

Y_{26}	Y_{27}	Y_{28}	Y_{29}	Y_{30}	Y_{31}
14.9	15.7	15.1	13.2	12.0	11.3

Arrange the ordinates as

	10.8	12.9	13.4	11.7	7.1	10.5	11.1	9.8	7.3	4.0	1.8	1.4	2.4	1.3	0.9	1.5	3.2
		11.3	12.0	13.2	15.1	15.7	14.9	13.0	9.6	7.2	6.9	6.8	5.8	5.5	4.0	2.7	
p	10.8	24.2	25.4	24.9	22.2	26.2	26.0	22.8	16.9	11.2	8.7	8.2	8.2	6.8	4.9	4.2	3.2
q		1.6	1.4	-1.5	-8.0	-5.2	-3.8	-3.2	-2.3	-3.2	-5.1	-5.4	-3.3	-4.2	-3.1	-1.2	

Rearrange p series as

	10.8	24.2	25.4	24.9	22.2	26.2	26.0	22.8	16.9
	3.2	4.2	4.9	6.8	8.2	8.2	8.7	11.2	
r	14.0	28.4	30.3	31.7	30.4	34.4	34.7	34.0	16.9
s	7.6	20.0	20.5	18.1	13.9	18.0	17.3	11.6	

and q series as

	1.6	1.4	-1.5	-8.0	-5.2	-3.8	-3.2	-2.3
	-1.2	-3.1	-4.2	-3.3	-5.4	-5.1	-3.2	
t	0.4	-1.6	-5.7	-11.3	-10.6	-8.9	-6.4	-2.3
u	2.8	4.5	2.7	-4.6	0.2	1.3	0	

Rearrange r series as

	14.0	28.4	30.3	31.7	30.4
	16.9	34.0	34.7	34.4	
v	30.9	62.4	65.0	66.1	30.4
w	-2.9	-5.6	-4.3	-2.7	

Rearrange u and v series as

	2.8	4.5	2.7	-4.6		30.9	62.4	65.0
	0	1.3	0.2			30.4	66.1	
k	2.8	5.8	2.9	-4.6	m	61.3	128.5	65.0
l	2.8	3.2	2.5		n	0.4	-3.7	

The results are finally tabulated as

Multiplier							
0.195		11.6			0.4		
0.38		17.3	-2.7		-1.6	2.8	
0.56		18.0			-5.7		
0.71		13.9	-4.3	-3.7	-11.3	5.8	2.8+2.5
0.83		18.1			-10.6		
0.92		20.5	-5.6		-8.9	2.9	
0.98		20.0			-6.4		
1	61.3 + 128.5 + 65.0	7.6	-2.9	0.45	-2.3	-4.6	3.2
Sum of	254.8	89.87	-12.13	-2.18	-37.30	+3.25	6.69
column	$32a_0$	$16a_1$	$16a_2$	$16a_4$	$16b_1$	$16b_2$	$16b_4$

$a_0 = 8$, $a_1 = 5.62$, $a_2 = -0.76$, $a_4 = -0.14$,

$b_1 = -2.33$, $b_2 = 0.20$, $b_4 = 0.44$

Therefore, the cell proliferation, y as a time function is given by

$$y = 8 + 5.62 \cos \frac{2\pi t}{24} - 0.76 \cos 2\frac{2\pi t}{24} - 0.14 \cos 4\frac{2\pi t}{24} - 2.33 \sin \frac{2\pi t}{24} + 0.20 \sin 2\frac{2\pi t}{24} + 0.44 \sin 4\frac{2\pi t}{24}$$

where t is the hour of the day. The value $a_0(=8)$ is the average number of cells actively involved in DNA synthesis during the 24 h period, where as y gives the number of such cells at any instant t (h) of the day.

The observed number of cells per fold section involved in DNA synthesis in 24 h period and the sum of the three oscillations (diurnal, semidiurnal and quarter-diurnal waves) in their number together with the steady value

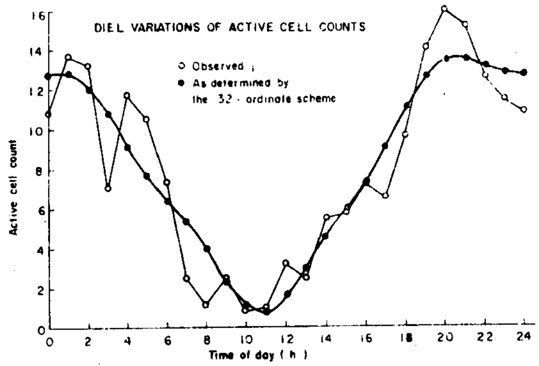

Fig. 1. Number of cells actively involved in DNA synthesis, of goldfish intestine : O = observed number of cells per fold section involved in DNA synthesis at different hours in a day. (From Nieto *et al.*, 1987 and • = Number of cells theoretically determined by the 32-ordinate scheme.

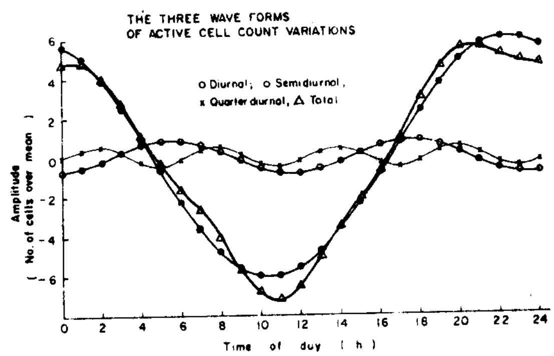

Fig. 2. The dial (cascade) wave forms of active cells.

(of the number of such cells) as determined by the 32-ordinate scheme are picturised in Fig. 1 and the cascade wave forms in Fig. 2. There is a good agreement between the observed and the scheme-determined value of the cell counts.

Calculations show that the amplitudes of the diurnal, semidiurnal and quarterdiurnal waves *i.e.* A_1, A_2 and A_4 are about 6.1, 0.8 and 0.5 respectively. The quarterdiurnal wave is the least. While the semidiurnal wave is just one and a half times large to the quarterdiurnal wave, the diurnal wave is twelve times large. Therefore, the diurnal wave is the most predominent variation of all the three.

Concluding Remarks

If the observed values are very haphazard, the 32-ordinate scheme is very useful as it takes enormous number of ordinates into analysis. In case the cyclic variation is referred to a lunar day (24.84) which is approximately 25 hrs, T stands for 25 h period which is to be divided into 32 equal intervals. The scheme is not limited to the daily variations of environmental parameters. It is applicable to any cascade type of rhythmic variations of any parameter.

It looks apparently odd to think of simple analytical designs, when the software computer technology is fast developing in the world. Nevertheless, simple schemes, as the present ones, find their usefulness in places where such advanced technological tools are not easily available. Moreover, it is not required to go to a computer, when the data to be handled are not voluminous.

Such simplified solutions obtained from the schemes may perhaps find their value in formulating simple prediction systems, especially when dealing with the daily variations of periodic functions. It may be concluded by quoting Prof. Naylor of the University of Liverpool (Naylor and Hartnoll, 1978) that juxtaposing the behavioural and physiological rhythms of marine plants and animals together with the ecological aspects of rhythmicity, would enhance the development of rhythm studies as a growth point in biological sciences.

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