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Macro Analytical Models - Surplus Production Models

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Production models are classified into two major groups namely Macro/ Global/ Synthetic models and Micro Analytical Models. Macro models are based on quite simple equations where both the state of the population and fishing activity are each described by a single variable. These models take into account only the interrelationship between observable inputs such as fishing effort and observable output which is the yield obtained from the fishery. Surplus production models are Macro analytical models.

Surplus production models are an important approach to the study of harvested populations dynamics. In surplus production models the stock is considered as a single unit of biomass and modeling is not based on any age structure, length structure or dynamics of the population in terms of growth and mortalities. Instead, in these models the entire stock, the fishing effort and the total yield obtained from the stock are studied and a relationship between these are established without considering any micro level details such as growth, mortality, age at first capture, mesh size effect etc. The objective here is to obtain optimum levels of effort, which gives the maximum yield that can be sustained over a long period. These models do not demand much data for the analysis and for this reason these models more popular. When reasonable estimates are available for the yield and corresponding fishing efforts over a period of time these models can be used for obtaining optimum levels of effort and corresponding yield estimates.

Change in biomass depends on recruitment, growth and mortality. This can be represented by the following equation

$$B_{t+1} = B_t + R_t + G_t - Z_t$$

where B_t is the biomass at time *t*, R_t is the weight of the new recruits into the fishery, G_t is the total increase in the weight of the animals due to growth and Z_t is the weight of the animals died during the period. Then production is given by

$$P_t = B_{t+1} - B_t = R_t + G_t - Z_t$$

The population is in equilibrium when production is zero.
When

$P_t = 0$, population is in equilibrium

$P_t > 0$, population is in surplus

$P_t < 0$, population is in depletion

The population may collapse when P_t goes beyond some values. Here biomass is a point time concept and yield or production is a period concept.

At given time *t*, under fishing activity f_t and population state B_t, the change in B_t is assumed to depend on population state and fishing activity. Hence the equation used commonly to define surplus production models is

$$\frac{dB_t}{dt} = g(f_t, B_t)$$

Different versions of this model are given by different workers, such as

1. Pella and Tomlinson

$$\frac{dB_t}{dt} = r\, B_t \left[1 - \left(\frac{B_t}{B_0}\right)^{m-1}\right] - q\, f_t\, B_t$$

2. Graham Schaefer's model

$$\frac{dB_t}{dt} = r\, B_t \left[1 - \left(\frac{B_t}{B_0}\right)^{2}\right] - q\, f_t\, B_t$$

3. Exponential model

$$\frac{dB_t}{dt} = r\, B_t \left[1 - \ln\left(\frac{B_t}{B_0}\right)\right] - q\, f_t\, B_t$$

Here B_0, r, m and q are parameters of the model which have to be estimated using data on yield and fishing effort.

In surplus production model the rate of increase in biomass is taken as a function of biomass itself so that the relative change is given by the equation

$$\frac{1}{B_t}\frac{dB_t}{dt} = f(B_t) - F_t \quad \text{where } F_t = q\, f_t$$

and F_t is the reduction in biomass due to fishing. When the production is surplus the relative change in biomass will be positive and it will be zero when the population is in the state of equilibrium and hence $f(B_t) = F_t$ at equilibrium.

Graham-Schaefer Model: In this model the first order differential equation is used to describe the rate of change of stock biomass B_t due to production. In the absence of fishing the rate of change in the biomass is assumed to be a function of current population size only.

That is

$$\frac{dB_t}{dt} = rB_t - \frac{r}{K} B_t^2$$

where B_t is the biomass at time *t*, *K* is the carrying capacity beyond which the population can not grow and *r* is the intrinsic rate of increase in stock per unit time. When fishing mortality is added to this model it becomes,

$$\frac{dB_t}{dt} = (r - F_t)B_t - \frac{r}{K} B_t^2$$

$$= \alpha_t \ B_t - \beta \ B_t^2$$

where $\alpha_t = (r - F_t)$, $\beta = \frac{r}{K}$ and F_t is the instantaneous rate of fishing mortality.

For a short period $(t = h, t = h + \delta)$ during which the instantaneous rate of fishing mortality F_t is constant, the solution of the differential equation gives

$$B_{h+\delta} = \begin{cases} \dfrac{\alpha_h B_h e^{\alpha_h \delta}}{\alpha_h + \beta B_h e^{\alpha_h \delta - 1}} & when\, \alpha_h \neq 0 \\ \dfrac{B_h}{1 + \beta\, \delta\, B_h} & when\, \alpha_h = 0 \end{cases}$$

and yield during the same period denoted by Y_h is

$$Y_h = \int_{t=h}^{t=h+\delta} F_t \, B_t \, dt$$

and solution of this integral yields

$$Y_h = \begin{cases} \dfrac{F_h}{\beta} \ln[1 - \dfrac{\beta\, B_h (1 - e^{\alpha_h \delta})}{\alpha_h}] & when\, \alpha_h \neq 0 \\ \dfrac{F_h}{\beta} \ln[1 + \delta\, \beta\, B_h] & when\, \alpha_h = 0 \end{cases}$$

The estimated average biomass during this short period $(t = h, t = h + \delta)$ is given by

$$\bar{B}_h = \frac{Y_h}{F_h}.$$

The surplus production during this period $(t = h, t = h + \delta)$ is

$$P_h = B_{h+\delta} - B_h + Y_h$$

When yield is equal to surplus production, the population is in equilibrium.

Parameter Estimation

It is assumed that the yield Y_t at equal time periods *t=1,...,T* are available. The following notations and assumptions are made for estimation purpose.

B_t	:	Population biomass at start of time *t*
Y_t	:	Yield in biomass during time *t*
P_t	:	Surplus production during time *t*
F_t	:	Fishing mortality rate during time *t*,assumed to be proportional to fishing effort rate.
f_t	:	Fishing effort rate during time *t*
q	:	Catchability coefficient

$$F_t = q\, f_t$$
$$\alpha_t = r - F_t$$

Parameters to be estimated are *r*, *K*, *q* and the initial biomass B_1.

Algorithm for estimation

The estimation procedure is by minimizing an objective function. With some starting guess estimates of the parameters compute the initial biomass and project through time estimating the yield for each time point *t=1,...,T*. The procedure is then iterative leading to the general function minimization procedure with the function to be minimized is

$$f(r,K,q,B_1) = \sum_{t=1}^{T} [\log(Y_t) - \log(\hat{Y}_t)]^2$$

where Y_t is the actual yield and $\hat{Y}_t$ is the corresponding yield estimated according to the model. Fishing mortality can also be estimated from recorded yield using the equation

$$F_t = \begin{cases} \dfrac{\beta\, Y_t}{\ln[\dfrac{\beta\, B_t e^{\alpha_t - 1}}{\alpha_t} + 1]} & when\, \alpha_t \neq 0 \\ \dfrac{\beta Y_t}{\ln(1+\beta B_t)} & when\, \alpha_t = 0 \end{cases}$$

Pella and Tomlinson's Model: One problem with the Graham-Schaefer model is that the maximum sustainable yield *MSY* always occurs when the biomass is half the carrying capacity *K*. This is a direct consequence of the parabolic relationship between $\frac{dB_t}{dt}$ and B_t, which in turn follows from the linear relationship between per capita productivity and population size. Pella and Tomlinson (1969) proposed an alteration to the model for which uncouples B_{MSY} from *K*.

One form of this model is given by

$$\frac{dB_t}{dt} = \begin{cases} a\,B_t^n - b\,B_t & for\ 0 < n \le 1 \\ b\,B_t - a\,B_t^n & for\ n > 1 \end{cases}$$

Simple forms

1. The simple representation of Schaefer model is

$$(Y_t / f_t) = a + b\,f_t$$

For this model the catch per unit effort is considered as a linear function of effort and the linear relationship has negative slope and positive intercept. Under this model the catch per unit effort will be maximum when

$$f_t = \frac{-a}{b}$$

The maximum sustainable yield (*MSY*) for the model is

$$MSY = \frac{-a^2}{4b}$$

and the corresponding effort is

$$f_{MSY} = \frac{-a}{2b}$$

When we have time series data on catch and effort by a linear regression of catch per unit effort (Y_t / f_t) (CPUE) on effort f_t, we can estimate the coefficients *a* and *b* and calculate MSY using this estimates.

2. In the model suggested by Fox, exponential relationship between *CPUE* and effort is assumed. The model is given by

$$Y_t / f_t = e^{c + d\,f_t} \text{ or equivalently } \ln(Y_t / f_t) = c + d\,f_t$$

This function will have maximum value for the yield when

$$f_t = \frac{-1}{d}$$

and the maximum value of yield (*MSY*) is given by

$$MSY = \frac{-1}{d} e^{c-1}$$

Using time series data on catch and effort through a linear regression of logarithm of catch per unit effort $\ln(Y_t / f_t)$ on effort f_t, we can estimate the coefficients *c* and *d* and calculate MSY using this estimates.

Example: Given in the following table are total catch and total effort in standard boat days for the shrimp fishery during the years 1969 to 1978. Estimate MSY and corresponding fishing effort (F_{MSY}) based on Schaefer model and Fox model.

Year (i)	Yield Y(i)	Effort f(i)	Y/f	ln(Y/f)
1969	546.7	1224	446.65	6.102
1970	812.4	2202	368.94	5.911
1971	2493.3	6684	373.03	5.922
1972	4358.6	12418	350.99	5.861
1973	6891.5	16019	430.21	6.064
1974	6532.0	21552	303.08	5.714
1975	4737.1	24570	192.80	5.262
1976	5567.4	29441	189.10	5.242
1977	5687.7	28575	199.04	5.294
1978	5984.0	30172	198.33	5.290
Mean	4361.1	17285.7	305.20	5.7
Sd	2184.9	10657.0	97.60	0.3

Analysis for Schaefer Model

Regression Statistics	
Multiple R	0.8797919
R Square	0.7740338
Adjusted R Square	0.745788
Standard Error	51.85595
Observations	10

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept - (a)	444.45412	31.24687	14.22396	5.81E-07
f(i) - (b)	-0.008055	0.001539	-5.23484	0.000788

$$MSY = \frac{-a^2}{4b} = \frac{-(444.45412)^2}{4*(-0.008055)} = 6130923\ Kg = 6131\ tonnes$$

$$F\,MSY = \frac{-a}{2b} = \frac{-(444.45412)}{2*(-0.008055)} = 27588.6\ boat\,days$$

Analysis for Fox Model

Regression Statistics	
Multiple R	0.8847168
R Square	0.7827238
Adjusted R^2	0.7555642
Standard Error	0.1757386
Observations	10

	Coefficients	*Standard Error*	*t Stat*	*P-value*
Intercept	6.1499573	0.105895	58.07604	8.58E-12
f(i)	-2.8E-05	5.21E-06	-5.36838	0.000671

$$MSY = \frac{-\exp(c-1)}{d} = \frac{-\exp(6.1499573 - 1)}{-0.000028} = 6159160\ Kg = 6159\ tonnes$$

$$F\,MSY = \frac{-1}{d} = \frac{-1}{(-0.000028)} = 35721\ boat\,days$$