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On nonlinear procedure for obtaining length - weight relationship

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ABSTRACT

A comparative study of linear and non-linear parameter estimation procedures for allometric model describing the length - weight relationship is carried out. It is shown that the latter approach is the correct one from statistical point of view. It is demonstrated, by an illustration, that the proposed procedure may yield parameter estimates which are not only quantitatively different from the corresponding ones for linear estimation but also have a bearing on the biological interpretation.

Key words : Length-weight relationship, Linear procedure, Nonlinear procedure, Residual analysis

A large number of research papers published in various journals during last two decades or so are testimony to the fact that the studies on length - weight relationship have great relevance (Agarwal and Saksena 1979, Gille and Salomon 1994, Shanmugham 1994, Chakraborty 1995). Inferences derived from such studies have usually been used to describe the relative state of health of individuals (Pepin 1995). It is, therefore, essential to ensure that the model used to describe the length - weight relationship not only provides a strong fit to the data set but it also accurately describes the functional form of the relationship, thus preventing any possible erroneous conclusions.

The allometric model which is used to describe the length - weight relation involves parameters appearing in a non-linear manner. The usual practice followed is to get rid of the inherent non-linearity by converting it into a linear model using logarithmic transformation and then estimate the parameters of the linear model by the well-known method of least squares. The two main purposes of the present paper are to highlight the deficiencies in this procedure and describe the most appropriate estimation procedure valid for fitting such a non-linear model. The details are illustrated by finding out the relationship between length and weight of *Catla catla* fish species (Agarwal and Saksena 1979).

MATERIALS AND METHODS

It is well-known that from a biological point of view, the length L and weight W exhibit the following type of allometric relation $W = a\,Lb$, (1)

where a and b are the parameters to be estimated.

Linearized version

Logarithmic transformation of eq. (1) leads to a linear relationship between log W and log L. Assuming an additive error term e, we have

$$\log W = \log a + b \log L + e. \tag{2}$$

The usual procedure of estimation of parameters of eq. (2) is to employ the method of least squares. The details are as given by Devaraj (1983). This procedure has been used very widely due to its simplicity. However, from a statistical point of view, fitting of eq. (1) with an additive error term is not equivalent to fitting of eq. (2). Thus there is a need to discuss appropriate procedure valid for fitting allometric model with additive error term.

Nonlinear estimation

As in linear regression, in nonlinear case also, parameter estimates can be obtained by the method of least squares. However, minimization of residual sum of squares yields normal equations which are no longer linear in parameters. Since it is not possible to solve such nonlinear equations explicitly, the alternate approach is to obtain solutions by employing suitable non-linear iterative procedures. Four main methods of this kind are: (i) Linearization method, (ii) Gradient method, (iii) Levenberg-Marquardt technique, and (iv) Do not Use Derivatives (DUD) method. The details of these methods are given in SAS Users' Guide (1990). Basically, under all these iterative techniques, by starting with a good initial guess of the unknown parameters, we try to minimize the residual sum of squares through the vector of initial guesses until the relative reduction between successive residual sum of squares is less than a very small

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predefined positive value, say 10^{-8}. PROCNLIN facility of SAS package can be used to fit the non-linear allometric model by any one of these 4 iterative techniques. In the present situation, based on their relative merits and demerits, the most appropriate procedure will be selected for further analysis. It may be mentioned that other standard software packages like SPSS also have facilities to fit nonlinear models by using nonlinear estimation procedures.

Having obtained the parameter estimates, the next step is to carry out 'Residual analysis' for verifying the underlying assumptions. The assumption of randomness may be examined by using 'Run test' while the assumption of normality may be examined by using 'Shapiro-Wilk test', if the sample size is up to 50 and D'Agostino test, if it exceeds 50 (D'Agostino and Stephens 1986). Finally, to judge the adequacy of the fitted model, we compute the statistic 'Root Mean Square Error (RMSE)' given by

$$RMSE = \left[\sum_{i=1}^{n} (W_i - \hat{W}_i)^2 / n\right]^{1/2}$$

Evidently, the lower the value of RMSE, the better is the fit.

Table 1. Fitting of the allometric model

Model	Linearized version	Non-linear
(i) *Parameter estimates*		
a	0.347	0.013
b	2.180	3.012
(ii) *Goodness of fit statistics*		
RMSE	370.170	88.648
(iii) *Residual analysis*		
Run test ($\|Z\|$)*	6.075	0.865
Shapiro-Wilk test*	0.782	0.978

*The tabulated value at 5% level for run test is 1.96. Further, the tabulated value at 5% level for Shapiro-Wilk test corresponding to n=43 is 0.943 while it is 0.916 for n=24.

Table 2. Asymptotic standard error and 95% confidence-interval of parameter estimates

Parameter	Estimate	Standard error	Asymptotic 95% confidence-interval	
			Lower	Upper
a	0.013	0.002	0.007	0.018
b	3.012	0.052	2.907	3.120

RESULTS AND DISCUSSION

As mentioned in the beginning, the allometric model is fitted to the Agarwal and Saksena (1979) data set by employing nonlinear estimation procedures. We have selected this paper simply for the reason that the entire data set is given there. The authors have fitted the following model

$$W = 0.347\ L^{2.180} \qquad (4)$$

by using the 'linearized version'. They have also concluded that the *Catla catla* fish species does not obey the general law of cubic growth. However, detailed analysis of their results indicates that the RMSE value comes out to be 370.170 and neither the assumption of randomness nor of normality of residuals is met (Table 1). This casts doubt about the validity of the results obtained. Further, it is worth pointing out that some of the observed weights in the data set correspond to same values of length (Agarwal and Saksena 1979). Therefore, instead of considering these as separate data points, it is more appropriate to take the average values of weights corresponding to equal lengths. Thus 24 observations remain for the analysis.

We have employed all the 4 nonlinear estimation procedures described earlier to the above data set comprising 24 data points. It is observed that the convergence did take place for the parameter estimates by each one of the 4 iterative techniques. However, the number of iterations, which is 8 in Levenberg-Marquardt method, is found to be the fewest. The summary statistics are given in Table 1. A perusal of this table indicates that the growth pattern of *Catla catla* fish species is in agreement with the cubic law, viz. b=3.012. This implies that there exists an isometric pattern of growth which is in contrast to the results obtained by Agarwal and Saksena (1979). Further, the lesser RMSE value of 88.648 also indicates the superiority of the nonlinear allometric fit over the corresponding fit by linearized version. Moreover, the assumptions of residuals, viz. randomness and normality, are satisfied for the non-linear fit, since the run test Z value 0.865 being less than the tabulated value of 1.96 at 5% level, is in the acceptance region; the Shapiro-Wilk test statistic value of 0.978, being greater than the tabulated value of 0.916 at 5% level, is also in the acceptance region.

Furthermore, asymptotic standard error and 95% confidence-interval of parameter estimates obtained by the non-linear estimation procedure are presented in Table 2. Perusal indicates that the standard errors of parameter estimates are quite low and the corresponding confidence-intervals are short.

In short, we have shown above, that the short cut of using the 'linearized version' for determining length-weight relationship, though very widely used, is beset with many serious problems. However, in the past, when computers and software packages were not so readily available, this appeared to be the only way out but now there seems to be hardly any justification for continuing to use this methodology. Whether the results obtained by this approach will differ marginally or substantially from the ones obtained by using the correct approach of 'nonlinear estimation' will depend solely on the data. Finally, the message emerging out of this paper is that 'nonlinear estimation procedures' should invariably be used for determining length-weight relationship.

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