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Determination of harmonic coefficients without the aid of conventional analytical schemes

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Abstract

Harmonic analysis converts periodic function into waves of primary frequency and higher harmonics. Analytical schemes, in the past, were developed to determine the harmonic coefficients. As all the waves at a time pass through each of the ordinates which are at their respective positions along the primary period, the arithmatic sum of the amplitudes put together at the time of passing through an ordinate is made equal to that ordinate, or atleast in the practical way, the sum of the amplitudes at each ordinate tends to be equal to that ordinate. This view makes it possible to find the harmonic coefficients from the selected ordinates. And thus, the coefficients could be directly determined without adopting any analytical scheme. The new method of detemining harmonic coefficients is applied to SST observations of the Bay of Port Blair. The SST observations were conducted from on board FORV *Sagar Sampada* during September 29th (18 Hrs) to 24 Hrs. on 30th September, 1988 at hourly intervals. The theoretically evaluated SST values from harmonic coefficients were found to be closer to the observed values.

A periodic function repeats itself periodically at regular intervals like the radiation received per unit area at the earth's surface from sun and sky in a day of 24 hours. Harmonic analysis is a means to convert the periodic function into a series of sine and cosine functions of time which are waves with primary frequency and the higher harmonics. The higher harmonics are numerical multiples of the primary frequency which is one. The importance of harmonic analysis lies in its nature of predicting the occurrence of a parameter, in advance as we know that the high tides and low tides are predicted much in advance of the year and printed in Tide Tables. The parameter y_t at any instant t within the primary period T is expressed as

$$y_t = a_o + a_n \cos n\,(2\pi t/T + b_n \sin n\,(2\pi t/T)$$

Where n = 1 is the primary wave, n = 2,3 .... gives higher harmonics. A_o is equilibrium (steady) value of the parameter like the water level it is calm without waves. The amplitude A_n of the wave having frequency n is obtained from

$$A_n = (a_n^{\ 2} + b_n^{\ 2})$$

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Analytical schemes

Different analytical schemes have been formulated from a set of ordinates (known) selected at equal intervals spread over the primary period. Among them, Runge's 6-ordinate scheme which he developed in 1903 (Joseph Lipka, 1918) was famous. Later in 1905 and 1913 he further developed 12-ordinate and 24-ordinate scheme. The higher the number of ordinates involved in the scheme, the more would be the accuracy, although it is at the loss of simplicity. Runge's analytical schemes generate primary wave and higher harmonics in succession without omitting any frequency in between.

However in nature sun-controlled parameters in atmosphere, oceanographic including biological parameters and ionosphere (D-layer) ionisation density showed in their dial variation the diurnal, semidiurnal and quarter diurnal variations, skipping off the one-third diurnal (frequency 3) wave. In order to meet this natural requirement, Murty (1978) developed 8-ordinate scheme and later 16-ordinate (Murty 1987) and 32-ordinate (Murty 1994) schemes. All the three schemes of Murty give the 1st, 2nd and the 4th frequencies only, unlike Runge's schemes. However, less number of ordinates are involved in Runge's 6-ordinate scheme and Murty's 8-ordinate scheme which are simple and lucid. Later involving, somehow or other, all the 24-ordinates for more accuracy Murty (1995) developed "modified 8-ordinate scheme". Irrespective of number of ordinates involved, all the Murty schemes derive the diurnal, semidiurnal and quarterdiurnal harmonics suitable to natural processes.

Amplitudes of waves on their passing through the ordinates

In the present paper a view is expressed that all the waves with their respective phase differences pass through each ordinate, like water waves pass through the wave-height indicating vertical pole fixed at a point on the way the waves pass through. The only difference is that all the harmonic waves pass through the ordinate at a time but not in different times, of course retaining their respective phase differences. In view of this, the arithmic sum of all the amplitudes with their phase differences will be equal to that ordinate at that instant. This view facilitates to solve the harmonic coefficients from the known values of ordinates by farming the required number of simultaneous equations. In this way we can avoid adopting any one of the analytical schemes. As a matter of fact, the analytical schemes avoid framing simultaneous equations and the present method avoids the analytical scheming system.

Master table to determine harmonic coefficients:

From 8-ordinates *in a primary period* uniformly spread over the 24 hour period *of the day (fundamental period)* the following Master Table is schematically formed.

In the above table, the 8-ordinates y_o, y_1,.........y_7 equidistanced at 3 hour interval in the primary period T (=24 hours) are chosen. Y_7 occupies 21st hour (315° in 360° cycle) corresponding to the primary period T(=24 hours). The next spacing at the same interval will be 24 hours (=360°). The cycle is complete at 24 hours. However, t = 0 is the same point as t = 24 hours of the cycle. In other words, the closing point in a cycle is the same as the starting point of

Table -1: ***Master Table***

Division No.n	0	1	2	3	4	5	6	7
Hour of the day t	0	3	6	9	12	15	18	21
Angle corresponding to t	0°	45°	90°	135°	180°	225°	270°	315°
Ordinate Yn	Y_0	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7
$a_o \cos 0(2\pi t/T)$	a_o	a_o	a_o	a_o	a_o	a_o	a_o	a_o
$a_1 \cos 1(2\pi t/T)$	a_1	$0.71a_1$	0	$-0.71a_1$	$-a_1$	$-0.71a_1$	0	$0.71a_1$
$a_2 \cos 2(2\pi t/T)$	a_2	0	$-a_2$	0	a_2	0	$-a_2$	0
$a_4 \cos 4(2\pi t/T)$	a_4	$-a_4$	a_4	$-a_4$	a_4	$-a_4$	a_4	$-a_4$
$b_1 \sin 1(2\pi t/T)$	0	$0.71b_1$	b_1	$0.71b_1$	0	$-0.71b_1$	$-b_1$	$-0.71b_1$
$b_2 \sin 2(2\pi t/T)$	0	b_2	0	$-b_2$	0	b_2	0	$-b_2$
$b_4 \sin 4(2\pi t/T)$	0	0	0	0	0	0	0	0

the cycle. At the intervals of division, sin 4 $(2\pi t/T)$ is either 180° or 360° which is always zero. Correspondingly the 4th frequent sin 4 $(2\pi t/T)$ is zero. The arithmetic sum of coefficients in each column of the master table is equal to the corresponding ordinate of the column. This logical assumption makes it possible to frame the necessary number of simultaneous equations to evaluate the harmonic coefficients.

Workout example

Observations on SST (Sea Surface Temperature) for 24 hour period taken at hourly intervals by using bucket thermometer on 29th and 30th September, 1988 in the Bay of Port Blair from on board FORV *Sagar Sampada* are used for working out the example (Shammi Raj *et. al.,* 1990). The period 24 hours is divided into 8 equal parts. The ordinate at each part is chosen. The two ordinates (SST values) one on either sides of each of the chosen ordinate are added to the central one and the mean of trio-ordinates is obtained (Murty, 1995). Thus the entire 24 hour SST values formed into 8-oridinates, $y_0, y_1.........y_7$. The eight ordinates are like 8 pillars to compel the 3 total wave heights together the steady value when passing thought the pillar would be equal to the pillar height. The mean value of the above eight ordinates is the steady (undisturbed) value on which the waves are measured. The data on SST and the trio-mean ordinates are presented in Table-2.

From the above equations, the coefficients can easily be obtained. Now the harmonic coefficients in each column of the master table are evaluated. The calculated eight ordinates from harmonics and the orginal values are depicted in graph (Fig.1)

The harmonic equation of the parameter, SST, y_t at any time t (hour of the day) is given by

$$y_t = a_o + a_1\cos1(2\pi t/24) + a_2\cos2(2\pi t/24) + b_1\sin1\,(2\pi t/24) + b_2\sin2\,(2\pi t/24)$$

With the values of coefficients y_t will

Table -2. *SST data and the 8-ordinates:*

t hrs	SST°C (ordinates)	Trio-ordinate mean	t hrs.	SST°C (ordinates)	Trio-ordinate mean
23	27.26		11	28.75	
24	27.56	$27.43=y_0$	12	28.96	$29.04=y_4$
01	27.46		13	29.40	
02	27.56		14	29.96	
03	27.65	$27.64=y_1$	15	30.16	$30.13=y_5$
04	27.70		16	30.26	
05	27.76		17	30.16	
06	27.96	$27.96=y_2$	18	30.06	$30.03=y_6$
07	28.16		19	29.86	
08	28.38		20	29.46	
09	28.55	$28.50=y_3$	21	29.06	$28.98=y_7$
10	28.56		22	28.41	

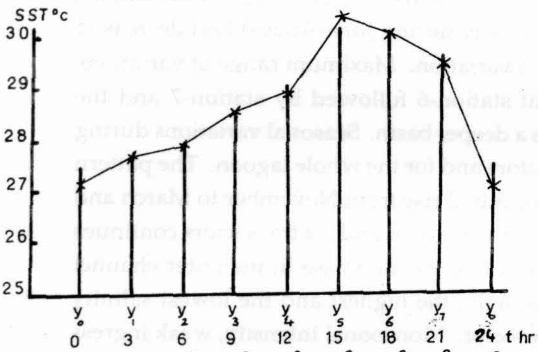

Fig.1. *Comparison of Selected Ordinates with those from harmonic coefficients*
columns - selected ordinates,
*** ordinates obtained from harmonic coefficients.*

become

$$y_t = 28.71 - 0.805 \cos(2\pi t/24) - 0.523 \cos2(2\pi t/24) - 0.243 \cos4(2\pi t/24) - 1.04 \sin1(2\pi t/24) + 0.14\ 2\sin2(2\pi t/24)$$

Amplitudes of the waves are

$A_1 = \pm(a^2_1 + b^2_1) = \pm 1.315$ diurnal wave

$A_2 = \pm 0.542$ (semidiurnal wave)

$A_4 = \pm 0.243$ (quarterdiurmal wave)

Harmonic analysis turns the periodic function into wave forms due to the characteristic nature of wave forms of cosine and sine functions which only are involved in harmonic series.

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