Madras Agric. J., 87(10-12): 597 - 600 October - December 2000

Prediction of maximum daily rainfall at Ootacamund

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Abstract : The maximum daily rainfall data for 30 years (1967-1996) of Ootacamund, Nilgiris, recorded by India Meteorological Department were collected from the Directorate of Statistics, Chennai. Two programs were developed in BASIC for Lognormal and Gumbel distributions. The theroetical values so obtained were in close agreement with the observed data except at the highest rainfall for both the distributions. *(Key Words : Rainfall, Probability distribution).*

Design engineers and hydrologists require maximum daily rainfall under different return periods for economic planning and design of small and medium hydraulic structures (small dams, bridges, culverts). Probability analysis is required to gather information on hydrological events that are governed by unknown physical law. Thus, lognormal distribution and Gumbel distribution were used to predict the maximum daily rainfall for Ootacamund.

Materials and Methods

Most hydrologic variables are assumed to come from a continuous random process and the historical sequence thereof is fitted with some common continuous distribution. Many probability distributions can be transformed by replacing the variate with its logarithmic value (Agarwal, M.C., Katiyar, V.S. and Rambabu, 1988). In the present analyses a transformed distribution (Lonormal distribution) and an extremal distribution (Gumbel distribution) were used.

Lognormal distribution

This is a transformed normal distribution in which the variate is replaced by its logarithmic value. This distribution represent the so-called law of Galton.

Its probability density is

$$p(x) = \frac{1}{\sigma y\, e^{y}\sqrt{2\pi}}\; e^{-(y-\mu y)^2/2\sigma y^2}$$

where $y=\ln x$, x is the variate, μy is the mean of y and σy is the standard deviation of y. This is a skew distribution of unlimited range in both directions.

Chow (1964) has derived the statistical parameters for x as

$$\mu = \exp(\mu y + \sigma y^2/2)$$

$$\sigma = \mu(e^{\sigma y^2}-1)^{1/2}$$

$$Cv = (e^{\sigma y^2}-1)^{1/2}$$

$$Cs = 3Cv + Cv^3$$

where σ is the standard deviation, Cs is the coefficient of skewness and Cv is the coefficient of variation.

Gumbel distribution

This is also known as Type I distribution. This distribution results from any initial distribution of exponential type, which converges to an exponential

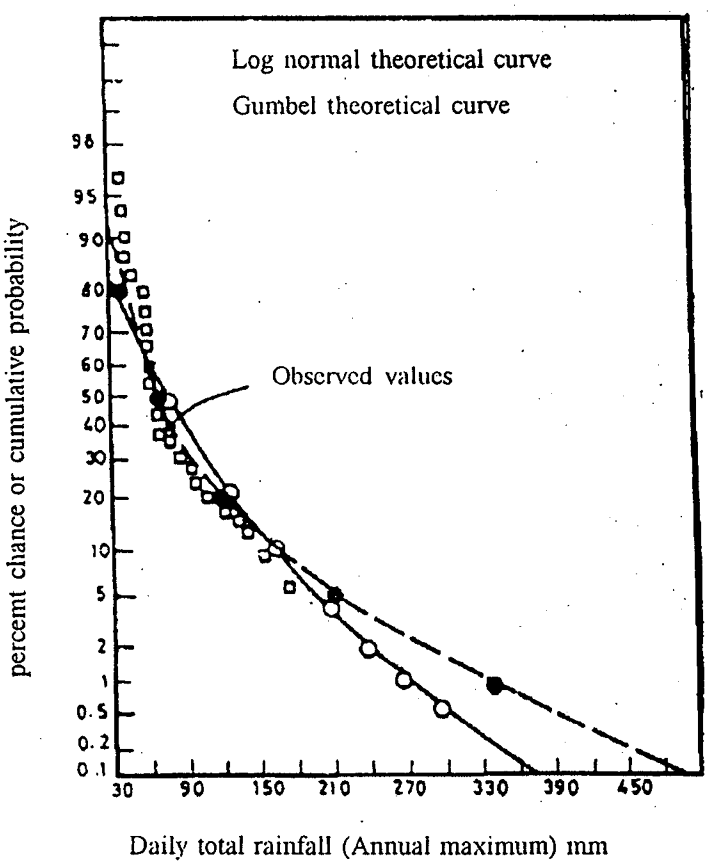

Fig 1. Frequancy analysis of maximum daily rainfall

Table 1. Theoretical probability of corresponding rainfall data for the lognormal distribution

Recurrence Interval years	Per cent chance or Probability	Frequency factor 'K'	C_vK	$x\sqrt{x}=1+C_vK$	X Thoretical Rainfall (mm)
1.0101	99	-1.0494	-0.84823	0.15177	12.823
1.0526	95	-0.9394	-0.75932	0.24068	20.334
1.25	80	-0.71	-0.573893	0.42611	36.001
2.0	50	-0.2753	-0.222525	-0.777475	65.686
5.0	20	0.5147	0.416032	1.416032	119.636
20.0	5	1.8547	1.499154	2.49915	211.145
100	1	3.7706	3.047776	4.04777	341.983
1000	0.1	7.4183	5.996212	6.99621	591.086
10,000	0.01	12.2378	9.891813	10.89181	920.213

Table 2. Theoretical probability of corresponding to rainfall data for the Gumbel distribution

Probability	Return period	Frequency factor K	$K\sigma_x$	$X=\bar{x}+K\sigma_x$
0.99	1.0101	-1.6408	-94.0916	9.6049
0.95	1.0526	-1.30567	-74.8736	9.6131
0.90	1.1111	-1.100286	-63.0959	21.3908
0.80	1.25	-0.8210947	-47.08567	37.40103
0.50	2.0	-0.1642798	-9.420625	75.066075
0.20	5.0	0.7194496	41.25683	125.7435
0.10	10.0	1.304555	74.8097	159.2964
0.04	25.0	2.043838	117.2038	201.6905
0.02	50	2.5922803	148.6543	233.141
0.01	100	3.136672	179.8723	264.359
0.005	200	3.6790789	210.9767	295.4634

function as x increases. The probability density of this distribution is

$$p(x) = \frac{1}{C} e^{-(a+x)/c-e^{-(a+x)/C}}$$

with $-\infty < x < \infty$, where x is the variate and a and c are parameters. The cumulative probability is

$$P(X \leq X) = e^{-e^{(a+x)}/C}$$

By the method of moments, the parameters have been evaluated as

$$a = \gamma C - \mu$$

$$\text{and } C = \frac{\sqrt{6}}{\pi} \sigma$$

where $\gamma = 0.57721$ a Euler's constant, μ is the mean, and σ is the standard deviation. This distribution has a constant coefficient of skewness equal to Cs = 1.139.

Recurrence Interval

The primary objective of the frequency analysis of hydrologic data is to determine the recurrence interval of the hydrologic event of a given magnitude x. The average interval of time within which the magnitude y of the event will be equal or exceeded once is known as recurrence interval return period or simply frequency, to be designated by T.

If a hydrologic event equal to or greater than x occurs once in T years, the probability $P(X \geq x) = 1/T$

$$\text{Hence, } T = \frac{1}{P(X \geq x)}$$

To calculate the return period the Weibull formula was followed. In this case the annual maximum daily total rainfall values were arranged in descending order of their magnitude. If n denotes the total number of years under consideration and m, the rank of the value from the highest value, then return period T was calculated as

$$T = \frac{n+1}{m}$$

Calculation of Expected Rainfall

The variate x of a random hydrological series may be represented by the mean $x \approx \mu$ plus the departure Δx of the variate from the mean, or

$$x = X + \Delta x \quad (1)$$

The departure Δx depends on the dispersion characteristic of the distribution of x and on the recurrence interval T and other statistical parameters defining the distribution. Thus, the departure may be assumed equal to the product of the standard deviation σ and a frequency factor K,

ie. $\Delta x = \sigma K$

The frequency factor is a function of the recurrence interval and type of probability distribution

Equation (1) can be expressed as

$$x = x + \sigma K \quad (2)$$

$$\text{or } x\sqrt{x} = 1 + CvK \quad (3)$$

where $Cv = \sigma\sqrt{x}$. The above equation was proposed by Chow as a general equation for hydrologic frequency analysis.

Chow (1964) has given the frequency factors for lognormal distribution. After selecting the probabilities in percent, corresponding to Cs and Cv values of the annual maximum daily total rainfall data, the frequency factors were taken from Chow's Table.

For Gumbel distribution the frequency factor was calculated using the formula proposed by Chow.

$$K = \frac{-\sqrt{6}}{\pi}\left[\gamma + \ln\left(\ln\frac{T}{T-1}\right)\right]$$

where γ is Euler's constant

and T is the Return period chosen

Thus knowing K values the expected rainfall was calculated using equation (3).

Results and Discussion

In the program for lognormal distribution, the number of years of data used for the analysis was given as input and the annual maximum daily total entered in the descending order of magnitude. Corresponding to the recurrence interval and Cs and Cv values, the frequency factors were entered from Chow's table.

In the program for Gumbel distribution, the number of years of data used were given as input and the annual maximum daily total entered in the descending order of magnitude. The selected recurrence intervals were entered in the program.

The theoretical probable maximum daily rainfall at 99, 95, 80, 50, 20, 5, 1, 0.1 and 0.01 per cent chances was the output obtained (Table 1) for lognormal distribution. For Gumbel distribution also the theoretical annual maximum daily total rainfall with corresponding assumed probabilities were obtained (Table 2). The observed rainfall data points closely lie around the theroretical probability curve when plotted in normal probability paper as shown in Fig 1 for both probability distributions except at the highest rainfall.

Any theoretical distribution is not an exact representation but only a description of the natural process which approximates the underlying phenomena and has proved useful in describing the observed historical data. It can be seen that except for the tail distributions, both the probability distribution models is a good fit to the data. Hence, any of the two distribution analysed may be used for prediction of annual maximum daily total rainfall at Ootacamund with reasonable accuracy.

References

Agarwal, M.C., Katiyar, V.S. and Ram Babu. (1988). Probability analysis of annual maximum daily rainfall of U.P. Himalaya. *India J. Soil Cons.* **16** (1) : 35-43.

Chow, V.T. (1964). Hand book of applied Hydrology. McGraw-Hill Book Company. New york **8** : (1-91).

(Received : July 1998 ; Revised : April 2001)