A METHOD OF ESTIMATION OF MORTALITY RATES FROM LENGTH SAMPLES

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ABSTRACT

Taking the probaib.'litj' distribution of age of fish as given by Ssentango and; Larkin (1973), and considering the size distribution as available from the d' to without taking recourse to the assumption of Marten (1978), a model is dtevelopsd to get an estimate of 'Z'. A table is appended to facilitate the estimation.

INTRODUCTION

In fish populationi studies, the total instantaneous mortality rate 'Z' is usually estimated from the knowledge of .the age distribution of the exploited population. In the absence of such knowledge, however, attempts have been made to estimate 'Z' in topical conditions where age determination is rather difficult. Ssentango and Larkin (1973), utilising the probability distribution (p.d.f.) of age t' have given a method of estimating 'Z'. Further, transforming this distribution to that in length, they have derived a simple method of estimating 'Z' on the basis of length measurements. But their method involves 'K' a parameter in the von Bertalaniffy's growth model. Hence it is necessary to estimate 'K' by some other methods like tagging experiments etc.

Marten (1978) has suggested a modification of the method of Ssentango and Larkin (1973) in which estimation of 'K' is avoided by postulating the following length growth model where L_t = Loc t when t lies between 0 and 1 and L_t = La for t greater than 1. In this mdei it is assumed that growth is constant after t = 1 which may not be necessarily true always.

PRESENT MODEL

It is generally known tiiat the exploited part of the population does not cover the enitire range of growth. Normally, the part of the population that is covered by exploitation may have the following growth forms, viz.,

$$l_t = l. + b(t - t.) \qquad \text{(I)}$$

or

$$It - l.c(^{M}.) \qquad <^{2)}$$

where t lies between t_0 and tj^ and b and c are constans, 1_0 and lj^ are sizes at $t = t_0$ and t = tj^j respectively, below and above which the sizes are under-represented. Since, any curve can be approximated by a series of straight lines the above assumptions may hold good in most of the cases. Even otherwise, whether the data conform to 'the above forms or not can be verified graphically by taking (to, lo) as the origin and plotting 'the successive modes against time.

The data available from the exploited fishes of India are found to closely coniform to the above growth forms. Some of the commercially important species exploited in India have a short duration of life span, and at their peak season of exploitation clear cut monthly modes could be discerned from the polymodal length distribution of catches. Here, though the initial age of entry of fish is not known the total period of life span covering all the distinct modes can easily be found out. With this background, in this paper a method is given for estimating 'Z' from length measurements utilising the p.d.f. as given by Ssentango and Larkin (1973).

ESTIMATION OF TOTAL MORTALITY

Case (1) We assume

$l_t = l_e + b\,(t-t_6)$ where t lies between t_0 and tj^j

and the p.d.f. of the age distribution in Oc'lv*) $^{t0\ \wedge e}$

$$p(t) = Ze^{'Zt}\ (e^{-Zt}o - e^{-Zt}M) \cdot' \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Hence, the mean length! is given by (integrating 1_1 p (t) in the interval $(t_0, tj^{\wedge})$)

$$f = l, + b\left(^{\wedge} - ae^{-Za} - \frac{e^{'Za)}\ (1-e^{"Za})}{Z}\right)' \quad \ldots\ldots\ldots\ldots (4)$$

where $a = t_M - t_0$ and $b = (1_M - l_c)/a$.

Thus we have,

$$('ivl\text{-}U \qquad \frac{Za}{e^{Za}-1} \qquad (5)$$

From this 'Za' can be obtained and since 'a' is known 'Z' can easily be estimated. The solution can be obtained by iteration method or the graphical method. In the graphical method the solution is given by the point of intersection of

$$y = Za(T-l_0)\ 0_M - l_o)^{-1}$$

and $y = 1 — Za\ (e^{Za}—1)^{4}$

Case (2): Here we assume

$$l_t = I.\ c\ (*\text{-}*.)$$

where t lies between to andtj^. This reduces to case (1) by applying logarithmic transformation.

EXAMPLES

Three examples are considered to test the validity of the method cited above and results are given in table 1.

TABLE 1. *Comparison of estimates of* 'Z'

	Example - 1		Example - 2	Example - 3
	Kavirando Gulf	Emin Pasha Gulf		
1c	34.5 cms	34.5 cms	125 mm	102.5 mm
>M	85.0 cms	85.0 cms	265 mm	177.5 mm
r	41.9 cms	47.4 cms	191 mm	136.7 mm
a	7 months	7 monithis	27 fortnights	one year
* Z	11.6 (11.5)	6.3 (6.5)	0.345 (0.345)	0.577 (0.57)
t	one year	one year	27 fortnights	one year

For the first example data are taken from Marten (1978), pertaining to bottom trawl samples of the cat fish *Bagrus docmac* from Kavirando Gulf and Emin Pasha Gulf in Lake Victoria, Bast Africa.

In the second example the data are the outcome of a laboratory experiment on the growth of *Pangasius pangasius* the cat fish, beitween December 1964 and February '1966 covering 28 fortnights. (Growth studies on *Pangasius pangasius* from the laboratory experiments. R. Mallikarjuna Rao, MSS.)

For the third example the length measurements of *Sardinella longiceps,* oil sardine, caught in Thamgu vala (Boat seine) operated near Cochin during 1976 are considered (V. Balan et al 1979).

(Z is estimate of Z; Figures in the brackets are the estimates obtained in the respective studies). In the example - 2 the experimeint was started with 24 numbers and at the end of which 17 survived with a survival rate of 17/24 = 0.7083, Z being 0.345 for 27 fortnights, which agrees well with the estimate obtained

TABLE 2. *Values of I—? \ in terms of* 'Za\
V IM-I. /

Za	0	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0		.4992	.4983	.4975	.4967	.4958	.4950	.4942	4933	4925
.1	.4917	.4908	.4900	.4892	.4883	.4875	.4867	.4858	.4850	.4842
.2	.4833	.4825	.4817	.4809	.4800	.4792	.4784	.4776	.4768	.4759
.3	.4750	.4742	.4734	.4725	.4717	.4709	.4701	.4692	.4684	.4676
.4	.4667	.4659	.4651	.4643	.4634	.4626	.4618	.4610	.4601	.4593
.5	.4585	.4576	.4569	.4660	.4552	.4549	.4536	.4528	.4519	.4511
.6	.4503	.4495	.4487	.4478	.4470	.4462	.4454	.4446	.4438	.4430
.7	.4421	.4413	.4405	.4397	.4389	.4381	.4373	.4365	.4356	.4348
.8	.4340	.4332	.4324	.4316	.4308	.4300	.4292	.4284	.4276	.4268
.9	.4260	.4252	.4244	.4236	.4228	.4220	.4212	.4204	.4196	.4188
1.0	.4180	.4172	.4164	.4156	.4148	.4141	.4133	.4125	.4117	.4109
1.1	.4101	.4093	.4086	.4078	.4070	.4062	.4054	.4046	.4039	.4031
1.2	.4023	.4015	.4008	.4000	.3992	.3984	.3977	.3969	.3961	.3954
1.3	.3946	.3938	.3931	.3923	.3915	.3908	.3900	.3892	.3885	.3877
1.4	.3870	.3862	.3855	.3847	.3839	.3832	.3824	.3817	.3809	.3802
1.5	.3794	.3787	.3779	.3772	.3764	.3757	.3750	.3742	.3735	.3728
1.6	.3720	.3713	.3705	.3698	.3691	.3684	.3676	.3669	.3662	.3654
1.7	.3647	.3640	.3633	.3625	.3618	.3611	.3604	.3597	.3589	.3582
1.8	.3575.	.3568	.3561	.3554	.3547	.3540	.3533	.3526	.3518	.3511
1.9	.3504	.3497	.3490	.3483	.3476	.3469	.3462	.3456	.3449	.3442
2.0	.3435	.3428	.3421	.3414	.3407	.3400	.3394	.3387	.3380	.3373
2.1	.3366	.3360	.3353	.3346	.3339	.3333	.3326	.3319	.3313	.3306
2.2	.3299	.3293	.3286	.3279	.3273	.3266	.3260	.3253	.3246	.3240
2.3	.3233	.3227	.3220	.3214	.3207	.3201	.3195	.3188	.3182	.3175
2.4	.3169	.3163	.3156	.3150	.3143	.3137	.3131	.3124	.3118	.3112
2.5	.3106	.3100	.3093	.3087	.3081	.3075	.3068	.3062	.3056	.3050
2.6	.3044	.3038	.3032	.3025	.3019	.3013	.3007	.3001	.2995	.2989
2.7	.2983	.2977	.2971	.2965	.2959	.2953	.2947	.2942	.2936	.2930
2.8	.2924	.2918	.2912	.2906	.2901	.2895	.2889	.2883	.2877	.2872
2.9	.2866	.2860	.2854	.2849	.2843	.2837	.2832	.2826	.2821	.2815
3.0	.2809	.2804	.2798	.2793	.2787	.2782	.2776	.2770	.2765	2759
3.1	.2754	.2749	.2743	.2738	.2732	.2727	.2721	.2716	.2711	.2705
3.2	.2700	.2695	.2689	.2684	.2679	.2673	.2668	.2663	.2658	.2653
3.3	.2647	.2642	.2637	.2632	.2627	.2621	.2616	.2611	.2606	.2601
3.4	.2596	.2591	.2586	.2581	.2576	.2571	.2566	.2561	.2556	.2551
3.5	.2546	.2541	.2536	.2531	.2526	.2521	.2516	.2511	.2506	.2502
3.6	.2497	.2492	.2487	.2482	.2478	.2473	.2468	.2463	.2459	.2454
3.7	.2499	.2444	.2440	.2435	.2430	.2426	.2421	.2417	.2412	.2407
3.8	.2403	.2398	.2394	.2389	.2384	.2380	.2375	.2371	.2366	.2362

TABLE 2. *{Continued)*

Za	0	.01	.02	.03	.04	.05	.06	.07	.08	.09
3.9	.2357	.2353	.2349	.2344	.2340	.2335	.2331	.2326	.2322	.2318
4.0	.2313	.2309	.2305	.2300	.2295	.2292	.2287	.2283	.2279	.2275
4.1	.2270	.2266	.2262	.2258	.2254	.2249	.2245	.2241	.2237	.2233
4.2	.2229	.2225	.2220	.2216	.2212	.2208	.2204	.2200	.2196	.2192
4.3	.2188	.2184	.2180	.2176	.2172	.2168	.2164	.2160	.2156	.2152
4.4	.2148	.2144	.2141	.2137	.2133	.2129	.2125	.2121	.2117	.2114
4.5	.2110	.2106	.2102	.209«	.2095	.2091	.2087	.2083	.2080	.2076
4.6	.2072	.2069	.2065	.2061	.2058	.2054	.2050	.2047	.2043	.2039
4.7	.2036	.2032	.2029	.2025	.2021	.2018	.2014	.2011	.2007	.2004
4.8	.2000	.1997	.1993	.1990	.1986	.1983	.1980	.1976	.1973	.1969
4.9	.1966	.1962	.1959	.1956	.1952	.1949	.1945	.1942	.1939	.1935
5.0	.1932	.1929	.1925	.1922	.1919	.1916	.1912	.1909	.1906	.1903
5.1	.1899	.1896	.1893	.1890	.1887	.1883	.1880	.1877	.1874	.1871
5.2	.1868	.1864	.1861	.1858	.1855	.1852	.1849	.1846	.1843	.1840
5.3	.1837	.1833	.1830	.1827	.1824	.1821	.1818	.1815	.1812	.1809
5.4	.1806	.1803	.1800	.1798	.1795	.1792	.1789	.1786	.1783	.1780
5.5	.1777	.1774	.1771	.1768	.1766	.1763	.1760	.1757	.1754	.1751
5.6	.1748	.1746	.1743	.1740	.1737	.1735	.1732	.1729	.1726	.1723
5.7	.1721	.1718	.1715	.1713	.1710	.1707	.1704	.1702	.1699	.1696
5.8	.1694	.1691	.1688	.1686	.1683	.1680	.1678	.1675	.1673	.1670
5.9	.1667	.1665	.1662	.1660	.1657	.1654	.1652	.1649	.1647	.1644
6.0	.1642	.1639	.1637	.1634	.1632	.1629	.1627	.1624	.1622	.1619
6.1	.1617	.1614	.1612	.1609	.1607	.1605	.1602	.1600	.1597	.1595
6.2	.1592	.1590	.1588	.1585	.1583	.1581	.1578	.1576	.1574	.1572
6.3	.1569	.1566	.1564	.1562	.1560	.1557	.1555	.1553	.1550	.1548
6.4	.1546	.1543	.1541	.1539	.1537	.1535	.1532	.1530	.1528	.1526
6.5	.1523	.1521	.1519	.1517	.1514	.1512	.1510	.1508	.1506	.1504
6.6	.1501	.1499	.1497	.1495	.1493	.1491	.1489	.1486	.1484	.1482
6.7	.1480	.1478	.1476	.1474	.1472	.1470	.1468	.1466	.1464	.1461
6.8	.1459	.1457	.1455	.1453	.1451	.1449	.1447	.1445	.1443	.1441
6.9	.1439	.1437	.1435	.1433	.1431	.1429	.1427	.1425	.1423	.1421
7.0	.1419	.1417	.1415	.1414	.1412	.1410	.1408	.1406	.1404	.1402
7.1	.1400	.1398	.1396	.1394	.1393	.1391	.1389	.1387	.1385	.1383
7.2	.1381	.1379	.1378	.1376	.1374	.1372	.1370	.1368	.1367	.1365
7.3	.1363	.1361	.1359	.1358	.1356	.1354	.1352	.1350	.1349	.1347
7.4	.1345	.1343	.1342	.1340	.1338	.1336	.1335	.1333	.1331	.1329
7.5	.1328	.1326	.1324	.1323	.1321	.1319	.1317	.1316	.1314	.1312
7.6	.1311	.1309	.1307	.1306	.1304	.1302	.1301	.1299	.1297	.1296
7.7	.1294	.1292	.1291	.1289	.1288	.1286	.1284	.1283	.1281	.1279

TABLE 2. *{Continued)*

Za	0	.01	.02	.03	.04	.05	.06	.07	.08	.09
7.8	.1278	.1276	.1275	.1273	.1271	.1270	.1268	.1267	.1265	.1264
7.9	.1262	.1260	.1259	.1257	.1256	.1254	.1253	.1251	.1250	.1248
8.0	.1247	.1245	.1243	.1242	.1241	.1239	.1237	.1236	.1234	.1233
8.1	.1231	.1230	.1228	.1227	.1255	.1224	.1223	.1221	.1220	.1218
8.2	.1217	.1215	.1214	.1212	.1211	.1209	.1208	.1207	.1205	.1204
8.3	.1202	.1201	.1199	.1198	.1197	.1195	.1194	.1192	.1191	.1190
8.4	.1188	.1187	.1185	.1184	.1183	.1181	.1180	.1178	.1177	.1176
8.5	.1174	.1173	.1172	.1170	.1169	.1168	.1166	.1165	.1164	.1162
8.6	.1161	.1160	.1158	.1157	.1156	.1154	.1153	.1152	.1150	**.1149**
8.7	.1148	.1146	.1145	.1149	.1143	.1141	.1140	.1139	.1137	**.1136**
8.8	.1135	.1133	.1132	.1131	.1130	.1128	.1127	.1126	.1125	**.1123**
8.9	.1122	.1221	.1120	.1118	.1117	.1116	.1115	.1113	.1112	.till
9.0	.1110	.1109	.1107	.1106	.1105	.1104	.1103	.1101	.1100	.1099
9.1	.1098	.1097	.1095	.1094	.1093	.1092	.1091	.1089	.1088	.1087
9.2	.1086	.1085	.1084	.1082	.1081	.1080	.1079	.1078	.1077	.1075
9.3	.1074	.1073	.1072	.1071	.1070	.1069	.1068	.1066	**.1065**	.1064
9.4	.1063	.1062	.1061	.1060	.1058	.1057	.1056	.1055	.1054	.1053
9.5	.1052	.1051	.1050	.1049	.1047	.1046	.1045	.1044	.1043	.1042
9.6	.1041	.1040	.1039	.1038	.1037	.1036	.1035	.1033	.1032	.1031
9.7	.1030	.1029	.1028	.1027	.1026	.1025	.1024	.1023	.1022	.1021
9.8	.1020	.1019	.1018	.1017	.1016	.1015	.1014	.1013	.1012	.1011
9.9	.1010	.1009	.1008	.1007	.1006	.1004	. 1003	.1002	.1001	.1000
10.0	.0999									

from (5). From the above table it is seen the estimate of Z obtained by the method suggested, closely agrees with those obtained in the respective studies. To facilitate estimation of 'Za' and hence 'Z\ a table is appended which gives the values of .—r for various values of 'Za' ranging from 0 to 10.

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