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LINEAR PROGRAMMING TECHNIQUE IN FISH FEED FORMULATION

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Nutrition plays a vital role in improving animal productivity. Good deal of work has been done in India on nutritional requirements of livestock and poultry. But studies on fish nutrition especially on the marine species are comparatively of recent origin. The Central Marine Fisheries Research Institute has been undertaking experimental work to estimate the digestibility coefficients of different feed stuffs and the nutritional requirements of selected fin and shell fishes. Experiments are also conducted to test the efficiency of different feed mixtures and to study their economics. In this paper a versatile tool called 'linear programming technique' has been discussed in relation to fish feed formulation.

Linear Programming

Today the word 'programming' is almost synonimous with computer programming which is purely an aid to computation such as in solving a set of equations or evaluating an expression. By itself a computer programme does not directly contribute anything to the development of the formulations leading to the set of equations or the derivation of the expressions. On the other hand linear programming is essentially a mathematical formulation for the determination of optimal solutions which do not violate certain specifications imposed. When several variables and specifications are involved the computations in getting optimal solutions are very heavy and an electronic computer facilitates quick and efficient execution of the computation scheme.

A simple example will give an insight into the linear programming model. Consider two feed ingredients to be combined in such a way that the mixture satisfies certain vitamin requirements and at the same time involves minimum cost.

Let x_1 and x_2 be the quantities required from the ingredients '1' and '2' respectively. It is stipulated that the mixture contains at least 'a' units of vitamin A, and 'c' units of vitamin C. Let ingredient '1' contains a_1 units of vitamin A per kg and c_1 units of vitamin C and the ingredient '2' contains a_2 units of vitamin A and c_2 units of vitamin C. If p_1 and p_2 be the respective prices per kg, the linear programming model can be written as

Minimise

$$p_1x_1 + p_2x_2$$

subject to the condition that

$$a_1x_1 + a_2x_2 \geq a$$

$$c_1x_1 + c_2x_2 \geq c$$

Also for meaningful solution

$$x_1 \geq 0 \text{ and } x_2 \geq 0$$

This is the standard form of a linear programming problem.

It consists of 3 parts namely (i) the function whose value is to be minimised (or maximised if it is a profit function) called the objective function (ii) structural constraints to take care of the minimum requirements and (iii) the non-negativity condition.

The formulation is called 'linear' because the expression to be minimised and the inequalities involve only variables multiplied by constants and added together. There are no x^2 term, 10 g x or any non-linear form of the variables.

Historically, the first problem in linear programming was formulated in 1941 by the Russian mathematician L. Kantorovich and the American economist F.L. Hitcheock, both working independently. A systematic way for arriving at optimal solution is provided by the 'simplex method' developed by the mathematician George Dantzig who published it in 1947. Charnes et al. (1953) Heady and Candler (1960), Hadley (1963), Gass (1964) and Loomba (1978) gave a full account of the principles of linear programming and the step-by-step calculations involved.

One of the early studies following a systematic approach in arriving at least-expensive feed mixtures was made by Waugh (1951). In India too linear programming techniques for evolving feed mixtures for livestock were attempted in the sixties and seventies (Jacob, 1972). However in the fishery field very little work has been done in the country on least-cost feeds meeting nutrient requirements. One reason was the lack of information on requirements of nutrients like protein and minerals for fish. With the work done in this direction at C.M.F.R.I. the information base has widened and it is felt that attempts could be made now to use the technique for formulation of fish feeds (Chandge, 1987; Gopal, 1986; Kalyanaraman and Paul Raj, 1984; Paul Raj, 1983; Paul Raj and Ali, 1982; Paul Raj and Thirunavukkarasu, 1987). The present note is more for introducing the technique to the nutritionists, some of whom may not be familiar with it, and for illustrating the procedure through case studies.

Illustration 1

A nutritionist proposes to mix two available ingredients such that the mixture contains at least 12 units of vitamin A and 15 units of vitamin C. Ingredient '1' contains 1 unit of vitamin A per kg and 2.5 units of vitamin C. Ingredient '2' contains 2 units of vitamin A and 1 unit of vitamin C. It costs 3.0 Rs. per kg for ingredient '1' and Rs. 4.0 per kg for ingredient '2'. Determine the minimum-cost feed mixture.

Let the mixture contains x_1 kg of ingredient '1' and x_2 kg of ingredient '2'. The linear programming model can be written as

Minimise

$3.0(x_1) + 4.0\ (x_2)$ (cost function)

Subject to the constraints

$1.0(x_1) + 2.0(x_2) \geq 12.0$ (vitamin A constraint)

$2.5(x_1) + 1.0(x_2) \geq 15.0$ (vitamin C constraint)

and the non-negativity conditions.

$x_1 \geq 0$ and $x_2 \geq 0$.

This being a two-variate case can be solved graphically. For this, consider the limiting cases of the constraints namely,

$$1.0(x_1) + 2.0(x_2) = 12$$

and $$2.5(x_1) + 1.0(x_2) = 15$$

By suitable substitution, the graphs of these two straight lines can be drawn (see Fig. 1). The area common to these lines away from the origin is called the feasible region because any point in that region satisfies the specifications imposed. Now consider the cost function. Give the total cost a zero value and also a convenient value, say, Rs. 20. The lines can then be drawn as in the previous case on the same graph. For different values of the cost a set of parallel lines result, called iso-cost lines. An iso-cost line is the locus of all points (combinations of x_1 and x_2) which result in the same cost. It may be noted that as the iso-cost lines move away from the origin the cost also increases. We need to concern only that point of the iso-cost line which just touches the feasible region. The co-ordinates of that point gives the optimum solution as it is a point in the feasible region and at the same time involves only the minimum cost. If the cost line is moved up, the cost increases and if it is moved down it will not be in the feasible region. Thus the co-ordinates of the point 'A' namely 4.5 kg of ingredient '1' and 3.7 kg of ingredient '2' provide the optimum combination of the inputs (Fig. 1). Substituting in the cost equation the minimum cost works out to Rs. 28.7.

In the above case as there are only two constraints (apart from the non-negativity conditions) the optimum point is obvious from Fig. 1, namely 'A' the intersection point of the two constraint lines. (The intersection points with the axes are not considered here for simplicity). If there are three constraints there would be two intersection points (A_1 and A_2 in Fig. 2) and unless the iso-cost lines are drawn and shifted away from the origin towards the feasible region it would not be possible to decide on the optimal point (A_1 in the present case). It can be proved that the optimal point of a linear programming problem will always lie on the boundary of the feasible region.

With three variables the graphic solution becomes cumbursome. With more than three variables one may follow the 'simplex' method involving a systematic and step-by-step procedure to arrive at a feasible and at the same time optimal solutions (please see references).

Illustration 2

Consider formulation of a feed mixture for *P. indicus* with ingredients shown in the following table and subject to minimum nutrient contents. The quantity of the mixture to be prepared is 100 kg.

Nutrients \ Ingredients	Ground nut cake	Fish meal	Shrimp head meal	Rice bran	Minimum nutrient contents specified
Protein (%)	38	55	40	11	35
Gross Energy (Mcal/kg)	3.8	4.1	3.0	3.2	3.2
Calcium (%)	0.25	4.50	10.00	0.06	1.0
Phosphorus (%)	0.65	2.50	2.20	1.50	1.5
Price (Rs/kg)	3.00	6.00	1.00	1.50	Minimise cost

Let x_1, x_2, x_3 and x_4 be the respective quantities in kg of groundnut cake, fish meal, shrimp head meal and rice bran required for the mixture. The minimum-cost linear programming model can be written as,

Minimise

$$3.00(x_1) + 6.00(x_2) + 1.00(x_3) + 1.50(x_4) \quad \text{(cost function)}$$

Under the constraints

$$0.38(x_1) + 0.55(x_2) + 0.40(x_3) + 0.11\ (x_4) \geq 35.0$$

(Protein requirement)

$$3.8(x_1) + 4.1(x_2) + 3.0(x_3) + 3.2(x_4) \geq 320.0$$

(Energy requirement)

$$0.0025(x_1) + 0.045(x_2) + 0.10(x_3) + 0.0006(x_4) \geq 1.0$$

(Calcium requirement)

$$0.0065(x_1) + 0.025(x_2) + 0.022(x_3) + 0.015(x_4) \geq 1.5$$
(Phosphorus requirement)

$$1.0(x_1) + 1.0(x_2) + 1.0(x_3) + 1.0(x_4) = 100$$
(Quantity requirement

$$x_1 \geq 0, \quad x_2 \geq 0, \quad x_3 \geq 0 \quad \text{and} \quad x_4 \geq 0$$
(Non-negativity requirement)

The computations involved in solving the above problem is quite heavy. But computer programme packages are available and with the aid of an electronic computer the solution can be obtained in a few minutes. The solution is not given in this note. Here the stress has been for the procedures for building up of the linear programming scheme utilising the relevant information base.

Concluding Remarks

Application of the linear programming technique in the field of fish feed formulation envisages the computation of a minimum-cost feed mixture meeting several specifications. The main tasks are to quantify the nutritional and other specifications, to fix the values of the coefficients for conversion of feeds into their nutrient equivalents and to ascertain the availability and price of feed ingredients.

In the examples given 'greater than' restriction alone has been used. But the scheme is highly flexible. One would like to specify the requirements in the form of a range with upper and lower limits rather than a single limit. Sometimes the feed ingredient can be given an upper or lower limit, for example, an upper limit can be fixed for an ingredient, say, fish meal, which may be available only in limited quantities. Similarly changes in the price regime can be introduced into the problem and solved without much additional computation. The technique is thus highly manoeuvrable and the nutritionist should take full advantage to investigate the varied types of alternatives for deciding on the best scheme for implementation.

One of the important assumptions made in the formulation of the linear programming problem is that of linearity. The expression to be optimised and the inequalities are assumed to be linear functions of the variables. The linear model employs an assumption of fixed prices and constant returns to scale. Under the linear model the total protein content for example, of the feed mixture is assumed to be the sum of the protein contents of the individual ingredients whatever be the proportion of the ingredients in the mixture. Thus no interaction is envisaged. Another assumption is that the coefficients such as the ones used for conversion of feed ingredients into their nutrient equivalents and also the prices of ingredients are known with certainty. These assumptions may be unrealistic for some situations and more sophisticated techniques like non-linear and stochastic programmings could be thought of. But within certain limits, for the problem of getting optimum feed mixtures the assumptions can be taken to be fairly reasonable.

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