A SIMPLE METHOD OF REPRESENTING DIEL VARIATIONS OF A PARAMETER IN THE FORM OF DIURNAL, SEMIDIURNAL AND QUARTERDIURNAL WAVES

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ABSTRACT

The observations on a fluctuating parameter over a diel period of 24 h are transformed into three cascade waves, namely the diurnal wave, semidiurnal wave and quarterdiurnal wave, which oscillate over the steady (mean) value of the parameter. Such a transformation of the varying parameter is effected by a very simple analytical method based on the choice of sixteen ordinates of the parameter equidistantly placed in the diel period.

INTRODUCTION

It is ironical that Fourier analysis (harmonics), which is the most useful analytical tool even in the modern computer analysis of time-series data, was discarded by the jury for its publication in a French journal on the ground that the paper lacks in rigorous proofs (Salvadori 1948). Later, based on the same Fourier analysis, many scientists developed simple analytical schemes to bring out time-series data in the form of harmonics. Among them, Runge's schemes of 6, 12 or 24 ordinates are very famous (Runge 1902, 1905, Salvadori 1948). The 6-ordinate Runge Scheme resolves the data into the 1st, 2nd and the 3rd harmonics over the steady (mean) value of the parameter over a period of fundamental or primary period. The 12-ordinate Runge Scheme presents the six harmonics from the 1st to the 6th in serial order. In a similar way, the 24-ordinate Runge Scheme gives all the harmonics up to the 12th one. There is no elimination of any harmonics in between the first and the last in each of the schemes developed by Runge.

Diel observations at regular intervals made over a period of 24 h of a fluctuating parameter in various fields, including oceanography and marine biology, can be expressed in the form of waves by adopting harmonic analysis. However, it is laborious to adopt harmonic analysis if our aim is only to drive the data to the point of diurnal, semidiurnal and quarterdiurnal wave forms (cascade waves). A reasonbly accurate but simple method is described here to sort out the data into such wave forms. The eight-ordinate scheme developed earlier (Murty 1978) though directly deals with such wave forms, the quarterdiurnal wave emerging out of the scheme is limited in its accuracy as it contains only the cosine factor, while the other waves are fully expressed by both sine and consine factors. This limitation of the eight-ordinate method is overcome by the choice of sixteen ordinates of the parameter equi-distantly placed in the diel period. Hence is the present scheme.

METHODOLOGY

Present the observed data in the graphical form with the parameter on the y-axis and the time on the x-axis. Consider a period of exactly 24 h on time axis, i.e., the period of a complete cycle, whatever be the starting hour. For txample, if the observations are started by 0700 hrs on one day, the end point of the cycle should be 0700 hrs the next day. If there is any difference between the ordinates corresponding to the beginning and ending of observations, these ordinates should be replaced by the mean of the two observations, so that the cycle of 24 h is complete.

Divide the total period T of the day (24 h) into 16 equal parts, starting from 00 hrs, as time is measured from midnight to midnight. This is done irrespective of the starting time of observations. (Remember that in a closed cycle the beginning point and the ending point are one and the same). After dividing the 24 h period into 16 parts, consider the 16 ordinates corresponding to these intervals, starting from the ordinate at 00 hrs. In this order let the ordinates y be represented with the suffixes 0 to 15. Thus, the chosen ordinates would be

$$Y_0, Y_1, Y_2 \ldots\ldots\ldots\ldots Y_{13}, Y_{14} \text{ and } Y_{15}.$$

Arrange and handle the ordinates in the following manner:

Y_0	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7	Y_8
	Y_{15}	Y_{14}	Y_{13}	Y_{12}	Y_{11}	Y_{10}	Y_9	
sum....p_0	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8
difference....	q_1	q_2	q_3	q_4	q_5	q_6	q_7	

Rearrange the p and q series as

p_0	p_1	p_2	p_3	p_4		q_1	q_2	q_3	q_4
p_8	p_7	p_6	p_5			q_7	q_6	q_5	
r_0	r_1	r_2	r_3	r_4	....sum....	t_1	t_2	t_3	t_4
s_0	s_1	s_2	s_3		..Difference..	u_1	u_2	u_3	

and the r series as

	r_0	r_1	r_2
	r_4	r_3	
Sum	v_0	v_1	v_2
Diff.	w_0	w_1	

After the above procedure, fill the following table as instructed therein with the multiplying numbers 0.38, 0.71, 0.92 and 1 in the first column of the table:

Multiplier							
0.38		s_3			t_1		
0.71		s_2	w_1		t_2	u_1+u_3	
0 92		s_1			t_3		
1	$v_0+v_1+v_2$	s_0	w_0	v_0-v_2	t_4	u_2	u_1-u_3
sum of column	$16a_0$	$8a_1$	$8a_2$	$8a_4$	$8b_1$	$8b_2$	$8b_4$

Multiply the terms in each row with the corresponding multiplier fixed in the left. After multiplication, vertically, add the terms (terms of each column) taking their sign into consideration. This sum in each column is equal to the amount of the coefficient given below the column. Thus, the coefficients a_0, a_1, a_2, a_4, b_1, b_2, and b_4 are determined.

THE WAVE FORMS

Let t be any hour of the day. The diurnal wave is given by $a_1 \cos \left(\frac{2\pi t}{T}\right) + b_1 \sin \left(\frac{2\pi t}{T}\right)$. The semidiurnal wave is given by $a_2 \cos 2\left(\frac{2\pi t}{T}\right) + b_2 \sin 2\left(\frac{2\pi t}{T}\right)$ and the quarterdiurnal wave by $a_4 \cos 4\left(\frac{2\pi t}{T}\right) + b_4 \sin 4\left(\frac{2\pi t}{T}\right)$. The coefficient a_0 represent the steady value of the parameter over which the waves ride with their respective phases. The amplitudes A_1, A_2 and A_4 of the diurnal, semidiurnal and quarterdiurnal waves respectively are given by

$$A_1 = \sqrt{a_1^2 + b_1^2},\quad A_2 = \sqrt{a_2^2 + b_2^2},\quad A_4 = \sqrt{a_4^2 + b_4^2}.$$

If α_n is the phase angle of the wave number n which is the number of harmonic, then the phase angle is given by

$$\tan^{-1} \frac{(b_n)}{a_n} = \alpha_n$$

The ordinate value y predicted from the wave forms at any time t is given by the algebraic sum of the waves together with the steady value a_0.

Therefore

$$Y = a_0 + a_1 \cos\left(\frac{2\pi t}{T}\right) + b_1 \sin\left(\frac{2\pi t}{T}\right)$$

$$+ a_2 \cos 2\left(\frac{2\pi t}{T}\right) + b_2 \sin 2\left(\frac{2\pi t}{T}\right)$$

$$+ a_4 \cos 4\left(\frac{2\pi t}{T}\right) + b_4 \sin 4\left(\frac{2\pi t}{T}\right)$$

EXAMPLE

The 24-hour observations of the phytoplankton cell counts (in lakhs | litre of sea water) made in the mudbank waters at Alleppey at 2-h interval, starting from 1800 hrs (Indian Standard Time) on 16th August 1975 and completing the cycle by 1800 hrs the next day (Mathew et al 1984) are treated for the wave analysis here. The observed values are presented in Fig. 1 from which the 16 ordinates in the required sequence are

y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7
2.3	1.9	1.9	2.1	1.9	4.0	5.2	5.0
y_8	y_9	y_{10}	y_{11}	y_{12}	y_{13}	y_{14}	y_{15}
3.2	2.0	2.0	2.3	2.1	3.2	2.8	2.1

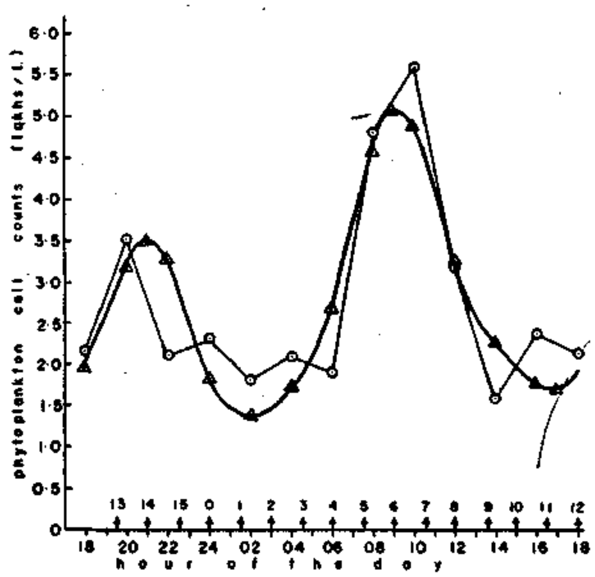

FIG. 1. Diurnal variations of phytoplankton cell count. (Circles: observed values; triangles: values from 16-ordinate scheme. The arrows along the x-axis indicate the location of the sixteen ordinates. The numbers above the arrows refer to the successive number of the 16-ordinates).

Arranging the ordinates as indicated early, we have

	2.3	1.9	1.9	2.1	1.9	4.0	5.2	5.0	3.2
		2.1	2.8	3.2	2.1	2.3	2.0	2.0	
p	2.3	4.0	4.7	5.3	4.0	6.3	7.2	7.0	3.2
q		—0.2	—0.9	—1.1	—0.2	1.7	3.2	3.0	

Arrange and handle the ordinates in the following manner:

Rearranging p and q series

	2.3	4.0	4.7	5.3	4.0
	3.2	7.0	7.2	6.3	
r	5.5	11.0	11.9	11.6	4.0
s	—0.9	—3.0	—2.5	—1.0	

	—0.2	—0.9	—1.1	—0.2
	3.0	3.2	1.7	
t	2.8	2.3	0.6	—0.2
u	—3.2	—4.1	—2.8	

and r series,

	5.5	11.0	11.9
	4.0	11.6	
v	9.5	22.6	11.9
w	1.5	—0.6	

The corresponding tabulation is:

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plier

From the above table,

0.38		—1.0			2.8		
0.71		—2.5 —0.6			2.3 —3.2—2.8		
0.92		—3.0			0.6		
1	9.5 + 22.6 + 11.9	—0.9	1.5	9.5—11.9	—0.2	—4.1	—3.2 + 28
Sum of column	$16a_0$	$8a_1$	$8a_2$	$8a_4$	$8b_1$	$8b_2$	$8b_4$

from the above table

$$a_0 = 2.75,\ a_1 = -0.73,\quad a_2 = 0.13,\quad a_4 = -0.30$$
$$b_1 = 0.38,\quad b_2 = -1.26,\quad b_4 = -0.05$$

Therefore y, the cell counts (lakhs | litre) at the time t (hrs) of the solar day of 24 h (T), is given by

$$y = 2.75 - 0.73 \cos\left(\frac{2\pi t}{24}\right) + 0.38 \sin\left(\frac{2\pi t}{24}\right)$$

$$+ 0.13 \cos 2\left(\frac{2\pi t}{24}\right) - 1.26 \sin 2\left(\frac{2\pi t}{24}\right)$$

$$- 0.30 \cos 4\left(\frac{2\pi t}{24}\right) - 0.05 \sin 4\left(\frac{2\pi t}{24}\right)$$

Fig. 2 represents the three cascade waves. The zero value of the wave amplitude refers to 2.75 (lakhs|litre) $= a_0$, (the steady value of the plankton count). The amplitudes of the diurnal, semidiurnal and quarterdiurnal waves are 0.82, 1.27 and 0.30 respectively. The semidiurnal wave is predominantly large. Its amplitude is twice that of diurnal wave. The quarterdiurnal wave is only about one-fourth of semidiurnal wave.

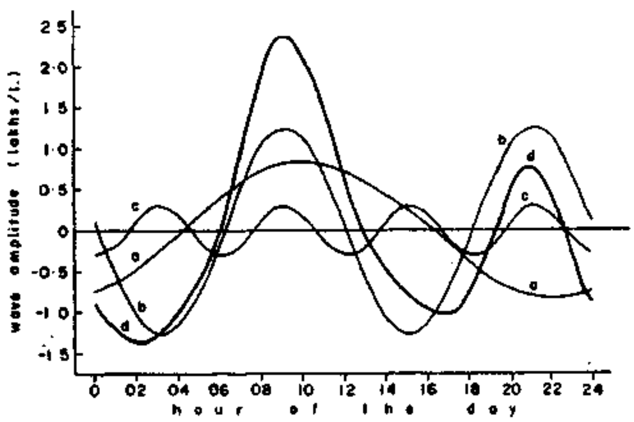

FIG. 2. Diel variations transformed into cascade waves. (a-diurnal wave; b - semidiurnal wave; c - quarter-diurnal wave; d - sum of the waves. The zeroline refers to a0 = 2.75 lakhs|l.).

CONCLUSION

In case the cycle is referred to a lunar day (24.84 h), which is approximately 25 h, T stands for 25 h period which is to be divided into 16 intervals starting from 00 hrs and the corresponding 16 ordinates are to be chosen. The method is not limited to the daily variations alone. It can be extended to all cascade type of rhythmic variations of any parameter. The analysis contemplates the first, the second and the fourth harmonics as they are the only harmonics in the diurnal, semidiurnal and quarter-diurnal variations in a complete cycle of a day. However, the coefficients a_3 and b_3 corresponding to the third harmonic can also be obtained from the same procedure. Two additional columns are required for a_3 and b_3 in the final table. One of the two additional columns