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THE TIDES OF THE OCEANS—A NEW PRESENTATION OF THE PRINCIPLE LUNAR TIDE M_2

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ABSTRACT

A realistic picture of the tides of world oceans requires considerable amount of data both from the coastal waters and from the open ocean. The data from the open ocean is seriously lacking at present. Three methods have been suggested to provide an image of the oceanic tides. The limitations of each of these methods have also been pointed out. Using the available informations, a picture of principle lunar tides - M_2 of the oceans has been given. This is an improvement over the picture presented by the author earlier.

INTRODUCTION

The question of the distribution of the oceanic tides in time and space can easily be answered, if the cotidal and cophase lines of the most important semidiurnal and diurnal tides could be presented properly. At present this is possible only partially. The reasons for this are well known. The tide-generating forces of the moon and the sun can be specified in all their details for the coastal areas, but not for the oceanic regions. Frictional effects, the reflexion of the tidal waves due to the complicated bottom topography and the density stratification in the open oceans, influence the propagation of the tidal waves too strongly.

Three methods are available at present to attack the problem of the oceanic tides and all these three have been applied here; the first is a theoretical one, the second is an empirical method and the third is a semi-empirical method. The theoretical method includes a straightforward integration of the system of the hydrodynamic equations. Besides the tide generating force as an external force, friction is also taken into account. Complete analyti-

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We are very happy to receive this article of late Prof. Dietrich from Dr. F. Schott. Those who have known Prof. Dietrich would appreciate how earnest he was in his promises and having promised to write for this special number he fulfilled it in his life time.

cal solutions of these equations, including the real bottom topography of the oceans have not been obtained up till now. Thus the approximation models of excessively generalized oceans have been developed by numerical integration. The empirical method, on the other hand, is based on the observations of the tides in the open ocean. This method can be applied if the harmonic constants of the most important tides are known from a grid of several hundred observation-points in the open ocean. However, the first measurement of the tides in the deep sea was carried out in 1965. Since then, only a few more records have been added. We should, however, hope that at least within the next decade many more records of the oceanic tides at about one month's interval will be available from many deep sea positions.

The large number of stations, from where harmonic constants have been published are restricted almost entirely to the coastal areas of the world ocean. Since a tidal wave changes its shape as it travels from the deep ocean towards the shelf and particularly towards the shallower part of the shelf near the coast, the known harmonic constants at these coastal positions are not valid for the open ocean, even closeby.

In these situations a semi-empirical method, such as the one applied by the author (Dietrich, 1944*a*) provides some solution. It cannot be the final answer as it is based on empirical data and theoretical results. There is no doubt that both these methods will be improved with time. The method, however, makes use of the harmonic constants from coastal stations, especially from the oceanic islands and from these the effect of the tidal waves on the shallow coastal regions can be estimated. Moreover, from the hydrodynamic expressions, the positions of the nodes and the amphidromies can also be derived. In this way, the world maps for the M_2, S_2, K_1 and O_1 tides have been developed. These maps are being used in many textbooks and manuals. Several other authors have arrived at similar results on tides (see Vallain, 1952; Bogdanov and Magarik, 1967). The purpose of this paper, therefore, is to reexamine the results available by applying the semi-empirical method which is limited to the M_2 tide. This seems necessary because although some progress has been made in recent years, neither the theoretical nor the empirical methods have given a satisfactory representation of the oceanic tides up till now.

THEORETICAL METHODS

Analytical solutions of the equations of motion have been provided by using grossly simplified models of the oceans (spheric biangle, constant water depth). These have given an insight into the number of amphidromies (Doodson, 1936 and 1938). More recent investigations have advanced our knowledge by integrating numerically the hydrodynamic equations—the so-called initial boundary-value problem from the work of Hansen (1948). It is thought to be a boundary-value problem because the solutions have to satisfy the kinematic boundary conditions along the coasts, i.e. the velocity component normal to the coast must disappear. The system of differential equations is changed into a system of difference equations and the ocean is covered by a grid of points, at which the unknown current velocities and sea level variations are calculated. If one starts with arbitrary initial values of this function, the current velocities and sea level variations can be calculated from the difference equations. As a result of friction, the influence of the arbitrary initial values is soon damped and the solutions are merely determined from the tidal forces.

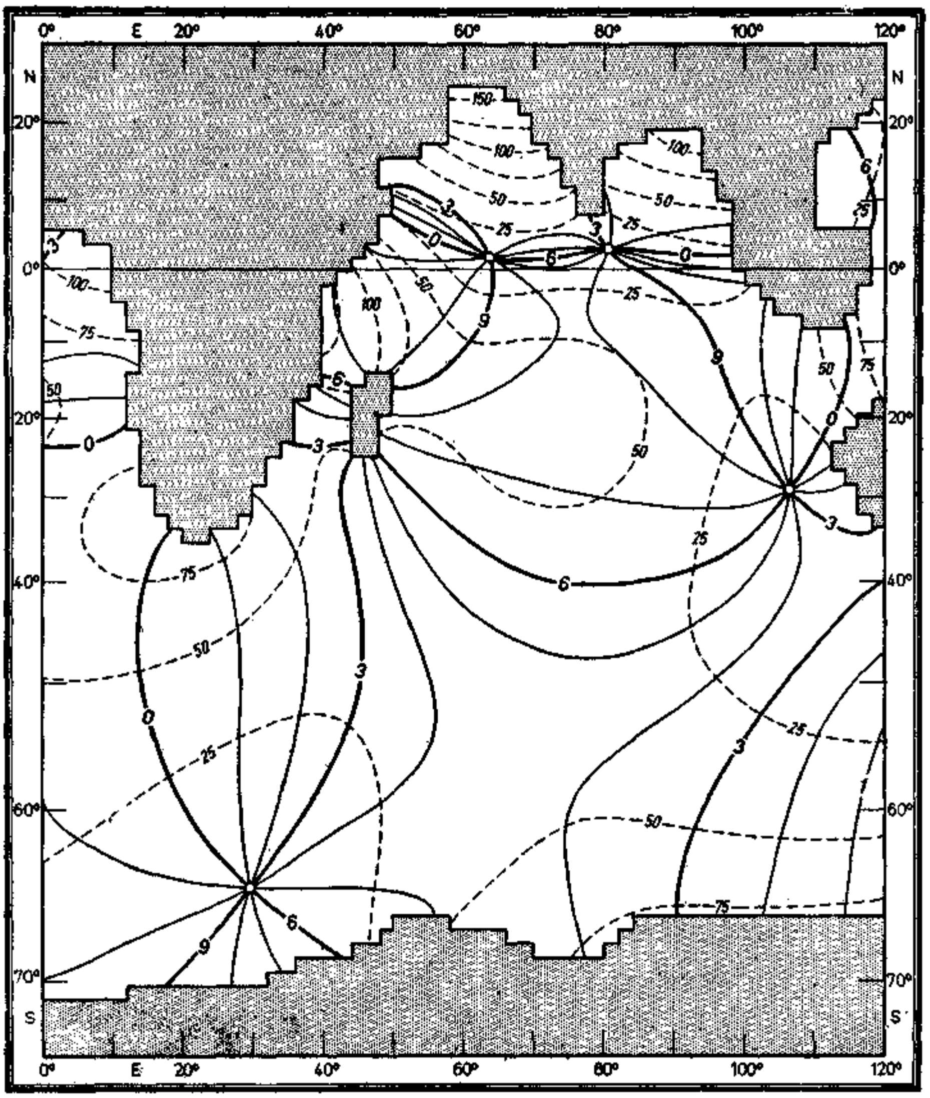

Fig. 1. Theoretically calculated distribution of corange lines (in centimetres) shown as broken lines, and cotidal lines, (in Greenwich lunar hours) shown as solid lines of the M2-tide. Part of the map taken from Pekeris and Accad, 1969.

Representations of the M_2 tide in the world ocean have been derived independently by Pekeris and Accad (1969) and by Zahel (1970) using the mathematical boundary-value method and without the knowledge of any tidal measurements. The model by Pekeris and Accad is based on a one-degree net (60 nm side-length at the equator), which means that a considerable amount of data for a large computer to analyse; Zahel uses a four-degree net (240 nm side length). Even the one-degree grid net is too wide for the calculation of the tides on the shelf-areas. Because the dispersion of tidal energy largely takes place on the shelf areas, it cannot be expected that this method can

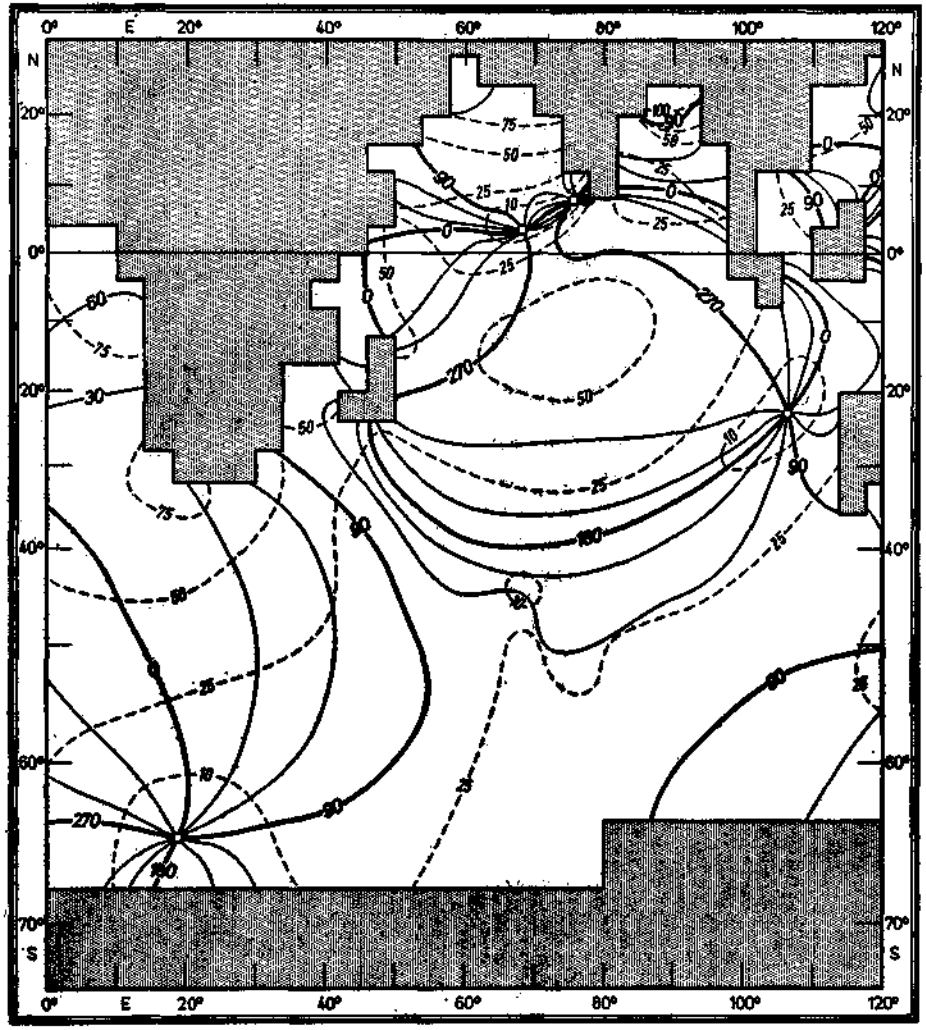

Fig. 2. Theoretically calculated distribution of corange lines (in centimeters) shown as broken lines and cotidal lines (in Greenwich lunar hours) shown as solid lines of the M2-tide. Part of the world map taken from Zahel, 1970.

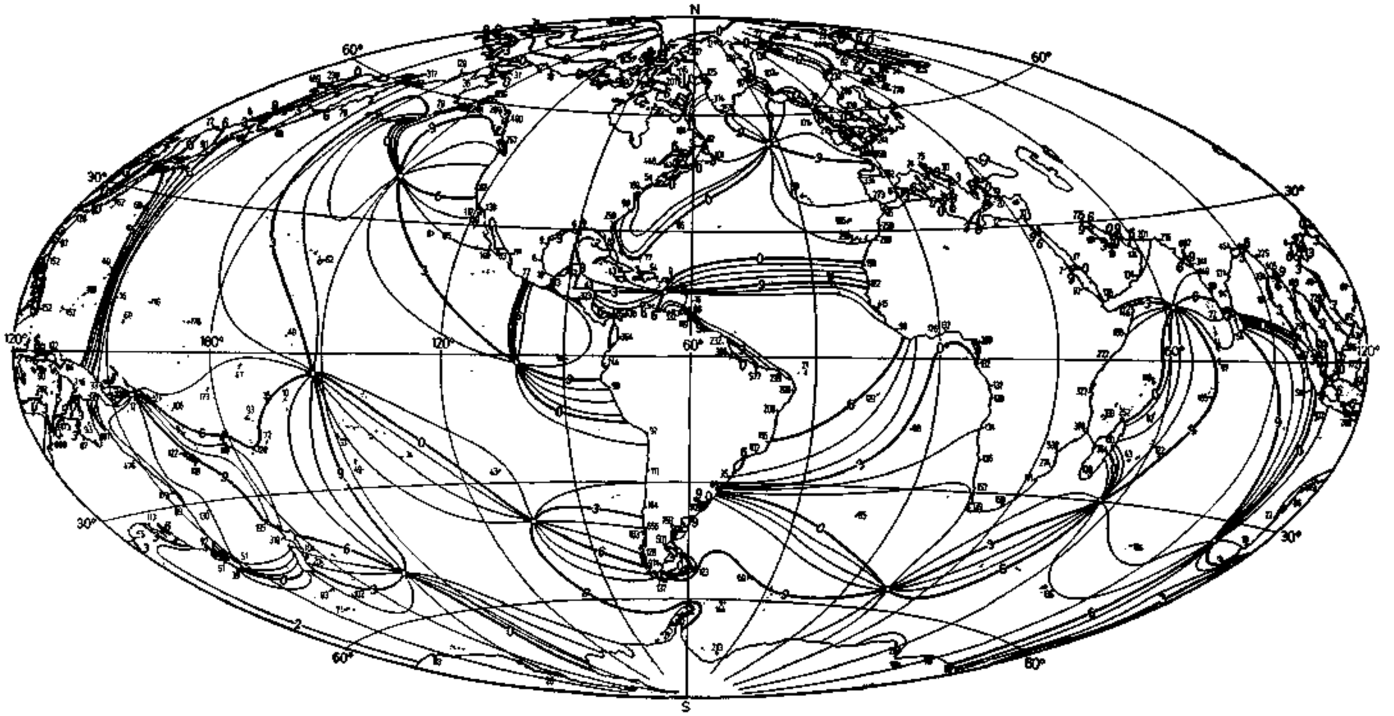

Fig. 3. Semi-empirically deduced distribution of cotidal lines (in Greenwich solar hours) of the M_2 - tides in the world Ocean. Numbers along the coast indicate observed mean spring tidal range of the semi-diurnal tides, $2\,(M_2 + S_2)$ (in centimeters). The former map published by Dietrich (1944) has been revised.

describe the tidal observations at the coasts. To do this, a better knowledge of the frictional conditions and of the transformation of energy of surface tides into internal tides as well as a smaller grid size for the calculations are necessary. For these, at present, the capacity of even the largest computers is insufficient. Hence, the results obtained by Pekeris and Accad and by Zahel have their own importance but these are mainly confined to describing the oscillation system of the semi-diurnal M_2 tide in the deep sea.

From the world maps based on these works, the various regions of the Indian Ocean can be studied (Figs. 1 and 2). The drawings of the cotidal and corange lines are such that the two figures can be compared, among each other things, with the semi-empirical representations of the phases shown in Fig. 3. The cotidal lines related to Greenwich have been shown in Figs. 1 and 2 in lunar hours, and in Fig. 3 in solar hours. These give a difference of 25 minutes per half a day. The corange lines in Figs. 1 and 2 give the double amplitude of M_2 in centimeters, but in Fig. 3 they include $2(M_2 + S_2)$, i.e., spring tidal range in centimeters. The two theoretically calculated representations of the M_2 tide in Figs. 1 and 2 are remarkably similar in phases as well as in ranges.

EMPIRICAL METHODS

These methods attempt to make tidal measurements in terms of sea level (pressure) registrations at many points of the ocean bottom at thousands of metres depth as well as in terms of current and density registrations in the water column above. The unusually high precision in the instruments used for measuring external pressures up to 600 atm, with an accuracy of sea level ± 1 cm, of current speed with ± 1 cm/sec, accuracy of water temperature with ± 0.02°C and of salinity with ± 0.02°/oo accuracy became possible only within the last few years, especially for the pressure registrations at great depths for at least 4 weeks continuously. The first deep-sea tide gauge for oceanic depths has been developed by Sutton *et al.* (1965), and subsequently by Eyries (1968), Snodgross (1968) and Filloux (1969). The instruments are very complicated and expensive; their launching depends on the weather conditions. The danger, however, for losing them is great. Therefore, it is not surprising to find that only a few analysed time-series are available. Among these, are 4 records from the gear 1968 and pertain to a profile perpendicular to the Californian coast extending about 1000 km into the open ocean. These records were made by Munk *et al.* (1970). The instruments during these observations had worked for several weeks at 4400 m depth. From the success of these recordings it is evident that empirical data collection of the oceanic tides on a global scale is possible and it is only a question of time before it materialises. It is also necessary to develop cheaper instruments so that many more countries and institutes are able to participate in a world wide programme. Such a programme has been arrived at by an international Working Group which was established by Scor in 1967 largely as a result of the initiative taken by Munk.

SEMI-EMPIRICAL METHODS

From the facts noted above it is clear that neither the theoretical nor the empirical methods provide a full description of the ocean tides. It is therefore necessary to make some derivations and these have been done by Dietrich (1944). The solution of the problem seems to be based on the harmonic constants of coastal areas obtained from the data collected from oceanic islands and on the system of amphidromies, which were derived from theoretical considerations (number, position, sense of rotation of the amphidromies). In the present report, the results of 1944 have been considerably improved because in all 6000 coastal stations with harmonic constants which were available at that time could now be increased by almost 1000 more, with highly valuable new records from the oceanic islands of the Pacific Ocean and from the Antarctica However, along the shelf regions the tides have been modified to such an extent, that the increase in the number of coastal observations has not provided a better insight into the tides of the open ocean than before. A striking example is of the Patagonian shelf, where the existencies of a special oscillation system with two amphidromies of semi-diurnal tides is similar to what has been found in 1944.

The results of the revised representations of the M_2-tide have been summarized in Fig. 3. The cotidal lines have been plotted and the system of the amphidromies includes all reliable harmonic constants. The range expressed as the spring tidal range $2(M_2 + S_2)$ has been given only for the selected coastal points. The tremendous influence of the shelf on the tidal range dose not permit to extend the isolines into the open sea from the coastal observations alone.

Since both theoretical and empirical methods do not allow for a realistic representation of the oceanic tides, it is very desirable that tidal observations from oceanic islands are carried out more intensively than before. These should last for at least one month.

COMPARISON BETWEEN THE RESULT OF THE THREE METHODS

A comparison between the theoretical and the semi-empirical method is possible from the presentation of the cotidal lines which have been shown in Figs. 1-3 for the region of the Indian Ocean. Some differences in the three figures are evident, but as far as the main features are concerned (number of amphidromies, their sense of rotation and their geographical position) the deviations between the three representations are not large. Instead of the theoretically calculated two amphidromies, only one has been confirmed by the empirical method from the North Indian Ocean. Probably the complicated bottom topography has been insufficiently taken into consideration in the theoretical method because of the coarse grid. Moreover, the semi-empirically derived amphidromy, south east of Madagascar, is very doubtful. Further observations in this region are necessary from oceanic islands such as Crozel and Prince Edward Islands.

A comparison of results using both the methods on M_2 tide is possible only for the profile off southern California. This has been shown in Table 1.

TABLE 1. *Comparison of phases G (hours) and amplitudes H (cm) of the M_2-tide in the eastern Pacific Ocean based on the observations of Munk et al.* (1970) *and from the theoretical calculations of Pekeris and Accad* (1969), *Zahel* (1970) *and from the semi-empirical results of the present author*

Position		Depth	Munk *et. al.*		Pekeris and Accad (1°grid)		Zahel		Dietrich	
ρ(N)	λ(W)	in m	G	H	G	H	G	H	G	H
31°02'	119°98'	3640	4.9	42.6	00.1	75	1.6	9.3	4.9	—
32°14'	120°51'	3700	5.2	42.5	00.0	63	4.8	9.0	5.1	—
27°45'	124°26'	4200	4.4	28.6	11.5	70	10.7	19.4	3.8	—
24°47'	129°01'	4400	3.3	18.8	11.7	65	10.9	29.8	2.8	—

The differences between the phases of the M_2-tides as observed by Munk *et al.* (1970) and those semi-empirically deduced by the present author are very small. However, the results obtained from the theoretical methods (Pekeris and Accad, 1969; Zahel, 1970) differ widely from the observed amplitudes and phases. This is because of the existence of an amphidromy in that area. It is of interest to note that the theoretical mehods provide only the basic knowledge on the oscillation system of the ocean tides but not on the local tides themselves.

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