J. Mar. Biol. Ass. India, **49** (2) : 197 - 205, July - December 2007

Vector time series modeling of marine fish landings in Kerala

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Abstract

Vector Autoregressive (VAR) type of models are used here to model and discover the relationships between landings of eight commercially important marine fish species/groups using quarterwise landings in Kerala during 1960-2005. Four different VAR models consisting of 4 time series each were developed and based on the significance of elements of parameter matrices in the model, constrained re-estimation was carried out to reduce the number of parameters. These models resulted in 16 individual models consisting of lagged terms of different landings time series and their relationships and influence on dynamic behavior of individual series were examined. Inter-dependence among component series was evident from these models as individual models contain lagged terms of other series.

Key words: Vector time series, VAR model, marine fish landings

Introduction

Modeling more than one time series together, termed as vector time series, is done to explore the relationship between the time series and to examine the structure responsible for their dynamic movement. Two major reasons for modeling and analysing more than one time series sequences together are (i) to understand the dynamic relationships among the different time series components, and (ii) to improve the accuracy of forecasts of one series by utilizing the information about that series contained in all other time series. Stergiou (1991) used vector autoregressions to describe and forecast sardine anchovy complex in the eastern Mediterranean. Using a VAR (6) model with two variables, he could explain 98% and 72% of variability in the catches of anchovy and sardines respectively. In this study, vector time series models of the type VAR (Vector Autoregressive model) were fitted to time series data on quarterwise landings of selected marine fish species/groups along the Kerala coast, southwest coast of India, to explore the relationship between the time series and to examine the structure responsible for their dynamic movement. The selection of the species/groups for the analysis were made based on their commercial importance, contribution towards total landings and biological aspects such as food and feeding habits, prey-predator relation etc. The species/groups selected for the study were the oil sardine, Indian mackerel, anchovies, lesser sardines, ribbonfishes, tuna, seerfish and elasmobranchs.

Being plankton feeders, the oil sardine, mackerel, lesser sardine and anchovies compete for food. Elasmobranchs, seerfish and ribbonfish are predators of oil sardine and mackerel. Lesser sardines and juveniles of ribbonfish feed on postlarvae of anchovies and adults of ribbonfishes feed on adults of anchovies. Seerfish (*Scomberomorous guttatus*) and tuna (*Auxis thazard*) also feed on anchovies. The lesser sardines are *Sardinella fimbriata*, *Sardinella albella* and *Sardinella sirm*. The important anchovies are *Stolephorus indicus*, *Stolephorus devisi*, *Stolephorus waitei* and *Stolephorus bataviensis;* the important seerfish are *Scomberomorus commerson*, *S. guttatus* and *S. lineolatus* and the important tunas are *Auxis thazard*, *Auxis rochei*, *Katsuwonus pelamis*, *Euthynnus affinis*, *Thunnus orientalis*, *Thunnus obesus* and *Thunnus tonggol*.

Materials and Methods

A set of k time series components is represented by a vector, $Z_t = (Z_{1t}, Z_{2t}, \ldots, Z_{kt})'$ termed as vector of time series. For stationary vector time series for all t where $\mu = (\mu_1, \mu_2, \ldots, \mu_k)'$ is the mean vector for the series and $E[(Z_t - \mu)(Z_{t+l} - \mu)] = \Gamma(l)$ is known as the cross covariance matrix of lag l for different $l = 0, \pm 1, \ldots$. If $\mathbf{V} = diag(\gamma_{11}(0), \gamma_{22}(0), \cdots, \gamma_{kk}(0))$ where $\gamma_{ii}(0)$ is the variance of the i^{th} component series Z_{it}, then $\rho(l) = \mathbf{V}^{-1/2}\Gamma(l)\mathbf{V}^{-1/2}$ is the cross correlation matrix at lag l. Estimate of elements of the lag l cross correlation matrix based on a sample of size T is computed as

$$\hat{\rho}_{ij}(l) = \frac{\sum_{t=1}^{T-l}(Z_{it} - \bar{Z}_i)(Z_{j(t+l)} - \bar{Z}_j)}{\sqrt{\left\{\sum_{t=1}^{T-l}(Z_{it} - \bar{Z}_i)^2\right\}\left\{\sum_{t=1}^{T-l}(Z_{jt} - \bar{Z}_j)^2\right\}}}$$

for $i,j = 1, \ldots, k; l = 0, \pm 1, \pm 2, \ldots$

where, $\bar{z}_i$ is the sample mean of the component i^{th} series.

A stationary vector time series Z_t with k components can be modeled by a vector autoregressive model of order p denoted by VAR(p), having the expression

$$z_t = \Phi_1 z_{t-1} + \cdots + \Phi_p z_{t-p} + \varepsilon_t .$$

where, $z_t = Z_t - \mu$; $\Phi_1, \Phi_2, \ldots, \Phi_p$ are $k \times k$ parameter matrices, $\varepsilon_t = (\varepsilon_{1t}, \ldots, \varepsilon_{kt})'$ are independently and identically distributed random innovation vectors having mean vector zero and constant covariance matrix Σ. The condition for stationarity of *VAR(p)* model, (Reinsel, 1993), is that the determinantal polynomial *det* $(I - \Phi_1 x - \ldots - \Phi_p x^p) = 0$ have all its roots out side the unit circle.

In the present study the parameter matrices of the fitted *VAR(p)* models were estimated by generalized least square method (Reinsel 1993) and estimate of innovation dispersion matrix was obtained as $\hat{\Sigma} = S_m / (n - \overline{kp + 1})$ where

$S_m = \sum_{t=p+1}^{T} \hat{\varepsilon}_t \hat{\varepsilon}_t'$ and $\hat{\varepsilon}_t = Z_t - \hat{\delta} - \sum_{j=1}^{p} \hat{\Phi}_j Z_{t-j}$. For large sample sizes under stationarity and Gaussian assumption, the approximate large sample distribution of $\hat{\phi}$ the estimate of $\phi = vec(\Phi_{(p)})$ is $N\left(\phi, \hat{\Sigma} \otimes \left(\tilde{X}'\tilde{X}\right)^{-1}\right)$ and this property was used to compute the standard errors of the estimates.

For selection of the order parameter p of vector autoregressive models, different order selection criteria were used. If $\tilde{\Sigma}_{(p)}$ is the maximum likelihood estimator of the innovation dispersion matrix Σ obtained by fitting a VAR(p) model to the data, then the Akike's *AIC* criterion (Akike, 1979) was calculated as

$$AIC(p) = \ln(|\tilde{\Sigma}_{(p)}|) + 2(pk^2 + k)/T$$

The other two criteria used are the Bayesian information criterion, *BIC* suggested by Schwarz's and the *HQ* criterion proposed by Hannan and Qunin. These criteria were calculated as

$$BIC(p) = \ln(|\tilde{\Sigma}_{(p)}|) + \ln(T)(pk^2 + k)/T \quad \text{and}$$

$$HQ(p) = \ln(|\tilde{\Sigma}_{(p)}|) + 2\ln(\ln(T))(pk^2 + k)/T$$

The orders that yield minimum value for these criteria were selected as the suitable order for the model.

Time series data on quarterwise landings of the selected species /groups in Kerala during 1960-2005 were transformed by taking a 4 point moving sum of natural logarithm and standardized for unit variance before fitting VAR models. Natural logarithm was taken to reduce the variability in individual series and 4 point moving sum was taken to remove seasonality. Four different vector time series models were attempted each consisting of four time series sequences. The first set was consisting of plankton feeders, the second group carnivores and the last two groups were consisting of two prey and two predator type combinations. For each set, suitable order for models were selected based on AIC, BIC and HQ criteria by considering orders up to lag 5 and parameters of VAR models were estimated by Generalized Least Square.

Results and Discussion

Model for oil sardine, lesser sardine, anchovies and mackerel: The three order selection criteria were computed using the vector time series consisting of oil sardine, lesser sardine, anchovies and mackerel as elements of the vector. The minimum AIC and HQ values for this vector series were for order 5 but the minimum BIC was for order 2. Since VAR(5) model consists of too many parameters the model considered was VAR(2) model.

The expression for the VAR(2) model is $y_t = \delta + \Phi_1 y_{t-1} + \Phi_2 y_{t-2} + \varepsilon_t$ where δ, Φ_1, Φ_2 are the parameters (first is a vector with 4 elements and others are square matrices of order 4 × 4 with 16 elements) and Σ is the innovation dispersion matrix (a symmetric square matrix of order 4×4 with 10 distinct elements). Estimates of parameter matrices and innovation dispersion matrix of the model are given in Table 1 along with estimates of standard errors in parenthesis. Since a VAR(2) model for this data set contains 36 elements in its parameter matrices, which are not all significant a constrained estimation by setting the non-significant elements to zero was made. These estimates along with the new estimates of standard errors and the estimated innovation dispersion matrix are given in Table 1. Individual models for the four components, 'oil sardine, lesser sardine, anchovies and mackerel' of the vector series are the first four models given in Table 2. From the individual models (series: 1 to 4) shown in Table 2, it can be seen that all the series depend on its past at lag 1 and also at lag 2 in some cases. This is partially due to the pre-processing of the quarterwise time series by taking a four point moving sum to remove seasonality in the data. Inter-dependence of component series is evident from most of the models as the individual series models contain in some cases lagged terms of other series.

Model for elasmobranchs, ribbonfish, seerfish and tuna: The order selection criteria AIC and HQ had minimum values for VAR(5) model whereas the BIC criterion had minimum value for VAR(1) model. The VAR(1) model, $y_t = \delta + \Phi_1 y_{t-1} + \varepsilon_t$ was then selected as the suitable model for this vector time series and model parameters estimated are given in Table 3 along with the estimates of standard errors and estimate of innovation dispersion matrix. Out of the 20 parameter elements in the model, only 8 were found significant and hence the model parameters were re-estimated by constraining the non-significant elements to zero. These estimates and the standard errors are also given in Table 3 with the estimate of innovation dispersion matrix. Individual models for the components, 'elasmobranchs, ribbonfish, seerfish and tuna' of the vector model (series: 5 to 8) are given in Table 2.

Model for oil sardine, anchovies, ribbonfish and seerfish: For the vector time series consisting of oil sardine, anchovies, ribbonfish and seerfish, the order selection criterion AIC had minimum value for VAR(5) and the HQ and BIC criterion had minimum values for VAR(2). Hence, the model selected for this data set is VAR(2), $y_t = \delta + \Phi_1 y_{t-1} + \Phi_2 y_{t-2} + \varepsilon_t$ and the parameters estimated for the model are given in Table 3. Out of the 36 parameter elements in the model, only 15 were found significant. Constrained estimation was carried out by setting the non-significant parameter elements into zeroes and the new estimates are also given in Table 4 along with the estimate of innovation dispersion matrix. The four models corresponding to the individual elements, 'oil sardine, anchovies, ribbonfish and seerfish' of the vector (series: 9 to 12) are given in Table 2 along with the percentage of variation explained by the models.

Model for elasmobranchs, lesser sardine, mackerel and tuna: Selection of the order for the

Table 1. *Estimate of constant vector δ and parameter matrices Φ_1 and Φ_2 with estimates of standard errors of elements in parenthesis for the VAR(2) model fitted for the vector series consisting of oil sardine, lesser sardine, anchovies and mackerel. The constrained re-estimates after setting the non-significant elements to zero and the estimate of innovation dispersion matrix Σ are also given*

$\hat{\delta}$	$\hat{\Phi}_1$				$\hat{\Phi}_2$			
0.8708 (0.4276)	1.4153 (0.0666)	-0.0279 (0.0454)	0.1132 (0.0633)	0.0015 (0.0643)	-0.4892 (0.0673)	0.0336 (0.0449)	-0.1451 (0.0644)	0.0101 (0.0630)
2.2839 (0.7196)	-0.0151 (0.1121)	1.0471 (0.0764)	0.0202 (0.1065)	-0.0360 (0.1081)	-0.0406 (0.1132)	-0.1714 (0.0756)	-0.0481 (0.1085)	0.0228 (0.1061)
0.5978 (0.5066)	-0.1484 (0.0789)	-0.0404 (0.5379)	1.0378 (0.0750)	0.0464 (0.0761)	0.1385 (0.0769)	0.0436 (0.0532)	-0.1036 (0.0764)	-0.0297 (0.0747)
0.6893 (0.4768)	0.0842 (0.0743)	-0.0005 (0.0506)	0.1811 (0.0706)	1.2135 (0.0716)	-0.1110 (0.0750)	-0.0108 (0.0501)	-0.1238 (0.0719)	-0.3180 (0.0703)
0.9651 (0.3198)	1.4177 (0.0637)	0	0	0	-0.4882 (0.0639)	0	-0.0285 (0.0198)	0
1.3183 (0.3564)	0	1.0663 (0.0721)	0	0	0	-0.1876 (0.0721)	0	0
0	0.0188 (0.0151)	0	0.9826 (0.0138)	0	0	0	0	0
0	0	0	0.0854 (0.0209)	1.2225 (0.0701)	0	0	0	-0.3247 (0.0686)

$\hat{\Sigma}$			
0.0668	0.0172	0.0030	0.0012
0.0172	0.1883	0.0073	0.0133
0.0030	0.0073	0.0966	0.0023
0.0012	0.0133	0.0023	0.0843

vector series consisting of elasmobranchs, lesser sardine, mackerel and tuna based on the three criteria yielded VAR(5) model for both AIC and HQ criteria and BIC criterion suggested VAR(2) model. For reasons of parsimony VAR(2) model, $y_t = \delta + \Phi_1 y_{t-1} + \Phi_2 y_{t-2} + \varepsilon_t$ was selected and model parameters estimated are given in Table 5. Only 16 out of the 36 parameter elements were found significant and hence the parameters were re-estimated by constraining the non-significant elements to zeros. The new estimates and the estimate of innovation dispersion matrix are also given in Table 5. Individual models representing the component series, 'elasmobranchs, lesser

Table 2. *Individual models derived from the four set of vector models fitted and the percentage of variation in each series explained by the models (y_{it} 's are the standardized 4 point moving sums of log transformed landings)*

No.	Series	Individual model	Variance explained (%)
	First set – Model for oil sardine, lesser sardine, anchovies and mackerel		
1.	Oil sardine	$y_{1t} = 0.9651 + 1.4177\, y_{1,t-1} - 0.4882\, y_{1,t-2} - 0.0285\, y_{3,t-2} + \varepsilon_{1t}$	93.32
2.	Lesser sardine	$y_{2t} = 1.3183 + 1.0663\, y_{2,t-1} - 0.1876\, y_{2,t-2} + \varepsilon_{2t}$	81.17
3.	Anchovies	$y_{3t} = 0.0188\, y_{1,t-1} + 0.9826\, y_{3,t-1} + \varepsilon_{3t}$	90.34
4.	Mackerel	$y_{4t} = 0.0854\, y_{3,t-1} + 1.2225\, y_{4,t-1} - 0.3247\, y_{4,t-2} + \varepsilon_{4t}$	91.57
	y_{1t} : Oil sardine, y_{2t} : Lesser sardine, y_{3t} : Anchovies and y_{4t} : Mackerel		
	Second set – Model for elasmobranchs, ribbonfish, seerfish and tuna		
5.	Elasmobranchs	$y_{1t} = 1.9398 + 0.9069\, y_{1,t-1} + \varepsilon_{1t}$	79.66
6.	Ribbonfish	$y_{2t} = 0.3153 + 0.9385\, y_{2,t-1} + \varepsilon_{2t}$	89.54
7.	Seerfish	$y_{3t} = 0.8896\, y_{3,t-1} + 0.1285\, y_{4,t-1} + \varepsilon_{3t}$	93.84
8.	Tuna	$y_{4t} = 0.0219\, y_{2,t-1} + 0.9849\, y_{4,t-1} + \varepsilon_{4t}$	96.12
	y_{1t} : Elasmobranchs, y_{2t} : Ribbonfish, y_{3t} : Seerfish and y_{4t} : Tuna		
	Third set – Model for oil sardine, anchovies, ribbonfish and seerfish		
9.	Oil sardine	$y_{1t} = 1.1096 + 1.4006\, y_{1,t-1} + 0.1842\, y_{4,t-1} - 0.4788\, y_{1,t-2} - 0.0497\, y_{2,t-2} - 0.1651\, y_{4,t-2} + \varepsilon_{1t}$	93.51
10.	Anchovies	$y_{2t} = 0.9995\, y_{2,t-1} + \varepsilon_{2t}$	90.26
11.	Ribbonfish	$y_{3t} = 1.2557\, y_{3,t-1} + 0.0519\, y_{2,t-2} - 0.3625\, y_{3,t-2} + \varepsilon_{3t}$	91.01
12.	Seerfish	$y_{4t} = 0.1173\, y_{1,t-1} + 0.0908\, y_{2,t-1} + 1.0990\, y_{4,t-1} - 0.1239\, y_{1,t-2} - 0.2118\, y_{4,t-2} + \varepsilon_{4t}$	94.47
	y_{1t} : Oil sardine, y_{2t} : Anchovies, y_{3t} : Ribbonfish and y_{4t} : Seerfish		
	Fourth set – Model for elasmobranchs, lesser sardine, mackerel and tuna		
13.	Elasmobranchs	$y_{1t} = 2.6872 + 1.1655\, y_{1,t-1} - 0.2939\, y_{1,t-2} + \varepsilon_{1t}$	81.69
14.	Lesser sardine	$y_{2t} = 1.1447\, y_{2,t-1} - 0.1160\, y_{2,t-2} + \varepsilon_{2t}$	79.81
15.	Mackerel	$y_{3t} = 1.7073 + 0.1109\, y_{1,t-1} + 1.2782\, y_{3,t-1} - 0.3075\, y_{4,t-1} - 0.1603\, y_{1,t-2} - 0.3885\, y_{3,t-2} + 0.3511\, y_{4,t-2} + \varepsilon_{3t}$	92.23
16.	Tuna	$y_{4t} = 0.0781\, y_{2,t-1} + 0.9971\, y_{4,t-1} - 0.0042\, y_{1,t-2} - 0.0674\, y_{2,t-2} + \varepsilon_{4t}$	96.19
	y_{1t} : Elasmobranchs, y_{2t} : Lesser sardine, y_{3t} : Mackerel and y_{4t} : Tuna		

sardine, mackerel and tuna' of the VAR(2) model (series: 13 to 16) are given in Table 2 along with percentage of variations explained by each model.

The above four sets of models give rise to 16 individual models (Table 2) for the eight series considered, giving two models for each series. For example, the two models 1 and 9 depict the dynamics of oil sardine series. Model 1 reveals

Table 3. *Estimate of constant vector δ and parameter matrix Φ_1 with estimates of standard errors of elements in parenthesis for the VAR(1) model fitted for the vector series consisting of elasmobranchs, ribbonfish, seerfish and tuna. The constrained re-estimates after setting the non-significant elements to zero and the estimate of innovation dispersion matrix Σ are also given*

$\hat{\delta}$	$\hat{\Phi}_1$			
3.1314 (0.8673)	0.8828 (0.0365)	0.0360 (0.0411)	-0.1558 (0.0786)	0.0513 (0.0883)
1.1410 (0.6352)	-0.0366 (0.0267)	0.9404 (0.0301)	-0.0015 (0.0575)	-0.0082 (0.0647)
-0.6319 (0.4782)	0.0388 (0.0201)	0.0152 (0.0227)	0.8622 (0.0433)	0.1223 (0.0487)
0.6143 (0.3835)	-0.0257 (0.0162)	0.0414 (0.0182)	0.0411 (0.0347)	0.9120 (0.0390)

1.9398 (0.6752)	0.9069 (0.0321)	0	0	0
0.3153 (0.1194)	0	0.9385 (0.0236)	0	0
0	0	0	0.8896 (0.0326)	0.1285 (0.0375)
0	0	0.0219 (0.0149)	0	0.9849 (0.0112)

$\hat{\Sigma}$

0.2034	0.0205	0.0239	0.0267
0.0205	0.1046	-0.0001	0.0127
0.0239	-0.0001	0.0616	0.0222
0.0267	0.0127	0.0222	0.0388

that, the oil sardine series is influenced by (i) its own past values at lags 1 and 2 and (ii) the past values of anchovies at lag 2. Since oil sardine and anchovies are considered again in the third set, (i) and (ii) are evident from model 9 also, with almost identical coefficients for the terms representing (i) and (ii). In addition, model 9 reveals that the oil sardine series is influenced by (iii) past values of seerfish at lags 1 and 2. From these results we can conclude that in addition to the effect of its own past values the oil sardine series is influenced by past values of anchovies at lag 2 and past values of seerfish at lags 1 and 2. The sign of the coefficients of the terms in the model indicates whether the influence is beneficial or not and the strength of the influence is indicated by the absolute value of the coefficient. Thus high landings of anchovies in one year is expected to marginally reduce the landings of oil sardine two years later whereas, the influence of seerfish landings on oil sardine landings is of mixed nature since the coefficients for lag 1 and lag 2 are of different signs with almost equal absolute values. When there is high catch of the predator (seerfish) in the previous year, causing reduction in their population size, the population of the prey (oil sardine) in the current year in the sea is expected to increase so that more oil sardine is available for exploitation resulting in increased landings of oil sardine. Similarly, if there are low landings of seerfish in the previous year and high landings in the year prior to that, we expect low landings of oil sardine in the current year. The same argument holds good for this situation also.

Individual models for lesser sardine series are models 2 and model 14. From these models it is evident that lesser sardine series depends on its own past values and it does not depend on any other series considered. From models 3 and 10 it can be seen that anchovies series is influenced by past values of oil sardine series at lag 1 and its own past values at lag 1. For

Table 4. *Estimate of constant vector d and parameter matrices F_1 and F_2 with estimates of standard errors of elements in parenthesis for the VAR(2) model fitted for the vector series consisting of oil sardine, anchovies, ribbonfish and seerfish. The constrained re-estimates after setting the non-significant elements to zero and the estimate of innovation dispersion matrix ∑ are also given*

$\hat{\delta}$	$\hat{\Phi}_1$				$\hat{\Phi}_2$			
1.0308	1.4012	0.1054	-0.0050	0.1755	-0.4761	-0.1409	-0.0136	-0.1574
(0.3297)	(0.0653)	(0.0631)	(0.0602)	(0.0805)	(0.0654)	(0.0652)	(0.0606)	(0.0777)
0.7086	-0.1353	1.0122	-0.0873	0.1461	0.1239	-0.1097	0.1221	-0.1152
(0.3879)	(0.0768)	(0.0742)	(0.0708)	(0.0948)	(0.0769)	(0.0768)	(0.0713)	(0.0914)
-0.1578	0.1095	-0.1011	1.2291	-0.1003	-0.0920	0.1620	-0.3188	0.0763
(0.3853)	(0.0763)	(0.0737)	(0.0703)	(0.0941)	(0.0764)	(0.0762)	(0.0708)	(0.0908)
-0.1054	0.1415	0.1774	-0.0565	1.0798	-0.1411	-0.1017	0.1020	-0.1964
(0.2978)	(0.0590)	(0.0570)	(0.0544)	(0.0727)	(0.0591)	(0.0589)	(0.0547)	(0.0702)

1.1096	1.4006	0	0	0.1842	-0.4788	-0.0497	0	-0.1651
(0.3189)	(0.0635)			(0.0782)	(0.0637)	(0.0310)		(0.0762)
0	0	0.9995	0	0	0	0	0	0
		(0.0022)						
0	0	0	1.2557	0	0	0.0519	-0.3625	0
			(0.0688)			(0.0119)	(0.0685)	
0	0.1173	0.0908	0	1.0990	-0.1239	0	0	-0.2118
	(0.0580)	(0.0227)		(0.0719)	(0.0581)			(0.0698)

$\hat{\Sigma}$				
	0.0649	0.0007	0.0040	-0.0044
	0.0007	0.0974	0.0060	0.0061
	0.0040	0.0060	0.0899	0.0036
	-0.0044	0.0061	0.0036	0.0553

increased landings of oil sardine we can expect slightly increased landings of anchovies in the next year. From models 4 and 15 representing mackerel series we see that apart from own past values at lags 1 and 2 the mackerel series is influenced by past values of anchovies at lag 1 and past values of both elasmobranchs and tuna at lags 1 and 2. An increased landing of anchovies is expected to cause a slightly increased landing of mackerel in the coming year and there is mixed type of influence of landings of elasmobranchs and tuna on future mackerel landings as the signs of coefficients are different for lags 1 and 2.

Table 5. *Estimate of constant vector δ and parameter matrices Φ_1 and Φ_2 with estimates of standard errors of elements in parenthesis for the VAR(2) model fitted for the vector series consisting of elasmobranchs, lesser sardine, mackerel and tuna. The constrained re-estimates after setting the non-significant elements to zero and the estimate of innovation dispersion matrix Σ are also given*

$\hat{\delta}$	$\hat{\Phi}_1$				$\hat{\Phi}_2$			
4.3007 (0.9354)	1.1472 (0.0745)	0.0358 (0.0727)	0.1269 (0.1071)	0.0362 (0.1749)	-0.3339 (0.0760)	0.0003 (0.0723)	-0.1636 (0.1069)	-0.1055 (0.1747)
0.5616 (0.9764)	0.0193 (0.0778)	1.0531 (0.0758)	-0.0256 (0.1118)	0.0228 (0.1826)	0.0244 (0.0794)	-0.1853 (0.0754)	0.0298 (0.1115)	-0.0345 (0.1823)
1.8493 (0.6283)	0.1329 (0.0501)	0.0379 (0.0488)	1.2541 (0.0719)	-0.3098 (0.1175)	-0.1940 (0.0511)	-0.0277 (0.0485)	-0.3557 (0.0718)	0.3414 (0.1173)
0.6554 (0.4311)	0.0413 (0.0335)	0.0919 (0.0335)	-0.0196 (0.0494)	1.0107 (0.0806)	-0.0729 (0.0350)	-0.0796 (0.0333)	0.0360 (0.0492)	-0.0489 (0.0805)
2.6872 (0.6861)	1.1655 (0.0681)	0	0	0	-0.2939 (0.0697)	0	0	0
0	0	1.1147 (0.0736)	0	0	0	-0.1160 (0.0735)	0	0
1.7073 (0.5743)	0.1109 (0.0464)	0	1.2782 (0.0662)	-0.3075 (0.1085)	-0.1603 (0.0471)	0	-0.3885 (0.0661)	0.3511 (0.1083)
0	0	0.0781 (0.0304)	0	0.9971 (0.0125)	-0.0042 (0.0077)	-0.0674 (0.0302)	0	0

$$\hat{\Sigma} = \begin{bmatrix} 0.1831 & -0.0082 & -0.0004 & 0.0231 \\ -0.0082 & 0.2019 & 0.0142 & 0.0000 \\ -0.0004 & 0.0142 & 0.0777 & 0.0159 \\ 0.0231 & 0.0000 & 0.0159 & 0.0381 \end{bmatrix}$$

From models 5 and 13, we find that the elasmobranch series is influenced by only its own past values at lags 1 and 2. Models 6 and 11 representing ribbonfish series reveals that it depends on its own past values at lag 1 and 2 and past values of anchovies at lag 2. Increase or decrease in the landings of anchovies is expected to cause slight increase or decrease in the landings of ribbonfish after two years. Models 7 and 12 represent the seerfish series and from these models it can be seen that apart from own past values at lags 1 and 2, the seerfish landings depends on past values of tuna at lag 1, past values of oil sardine at lags 1 and 2 and past values of anchovies at lag 1. Thus, higher landings of anchovies is expected to cause slightly increased landings of seerfish in the next year and there is mixed type of influence of oil sardine landings on future landings of seerfish since the coefficients representing lagged terms of oil sardine in the model are positive and negative respectively for lags 1 and 2. From models 8 and 16 representing the tuna series, we see that it depends on its own past values at lag 2, past values of ribbonfish at lag 1, past values of lesser sardines at lags 1 and 2 and past values of elasmobranchs at lag 2.

Acknowledgements

I am grateful to Dr. K. Kumarankutty, Professor and Head (Retired), Department of Statistics, University of Calicut for his valuable guidance in carrying out this research work. I express my gratitude to the Director, Central Marine Fisheries Research Institute for extending necessary facilities for the study. Also, I thank the Indian Council of Agricultural Research for the award of Senior Research Fellowship during the study.

References

Akaike, H. 1979. A Bayesian extension of the minimum AIC procedure of autoregressive model fitting. *Biometrika,* 66(2): 237-242.

Box, G. E. P. 1977. A canonical analysis of multiple time series. *Biometrika,* 64(2): 355-365.

Eakin, D. H. 1988. Estimating vector autoregressions with panel data. *Econometrika,* 56 (6): 1371-1395.

Grubb, H. 1992. A multivariate time series analysis of some four price data. *Appl. Statist.,* 41(1): 95-107.

Hannan, E. J. 1987. Rational transfer function approximation. *Statistical Science,* 2(2): 135-161.

Hsiao, C. 1979. Autoregressive modeling of Canadian money and income data. *JASA,* 74(367): 553-560.

Koch, P. D. and S. S. Yang. 1986. A method for testing the independence of two time series that account for a potential pattern in the cross-correlation function. *J. Am. Stat. Assn.,* 81(394): 533-544.

Pena, D. 1987. Identifying a simplifying structure in time series. *J. Am. Stat. Assn.,* 82(399): 836-843.

Reinsel, G. C. 1983. Some results on multivariate autoregressive index models. *Biometrika,* 70(1): 145-156.

Reinsel, G. C. 1993. *Elements of Multivariate Time series Analysis*. Springer-Verlag, New York. 253 pp.

Stergiou, K. I. 1991. Describing and forecasting the sardine anchovy complex in the eastern Mediterranean using vector autoregressions. *Fish. Res*., 11(2): 127-141.

Thompson, P. A. 1989. Multivariate time series projections of parameterrized age-specific fertility rates. *J. Am. Stat. Assn.,* 84(407): 689-702.

Tiao, G. C. 1981. Modeling multiple time series with applications. *JASA,* 76(376): 802-816.

Yamamo, T. 1981. Predictions of multivariate autoregressive moving average models. *Biometrika,* 68(2): 485-492.

Received: 24 January 2008
Accepted: 18 March 2008