

Application of Linear Programming in feed formulation

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In fish feed formulation we determine the types and proportion of ingredients to be mixed to produce a complete feed at possible low cost. The fish species in culture pond should be fed with a feed that imitates as far as possible the nutrient levels in their natural food. This is achieved to some extent by mixing different feed ingredients in a suitable way. Here, several factors need to be taken into consideration, as the feed should be nutritionally viable and economically feasible. The problem then is to determine exact optimum proportions of different feed ingredients, which meets the necessary nutritional requirements and have the minimum possible unit cost for the formulated feed. This is achieved through Linear Programming.

The word "Programming" is used here in the sense of "planning"; the necessary relationship to computer programming was incidental to the choice of name. The importance of linear programming derives in part from its many applications and in part from the existence of good general-purpose techniques for finding optimal solutions. Linear programming is about making the most of limited resources. It deals with maximizing / minimizing a linear function of variables subject to a set of linear constraints. Applications range from economic planning and environmental management to diet problems.

What is Linear Programming?

It is a Mathematical technique and not computer programming. It allocates scarce resources to achieve an objective. Linear Programming was pioneered by George B. Dantzig in World War II while being assigned to the U.S. military. He developed a workable solution called Simplex Method in 1947. A program can be thought of as a set of blocks, or activities, of different shapes that can be fitted together according to certain rules. Before linear programming and the simplex method were invented, it was not possible to computationally determine the best combination.

Requirements of a Linear Programming Problem

- Must seek to **maximize** or **minimize** (put together known as **Optimize**) some quantity (the objective function)
- Presence of restrictions or constraints – limits ability to achieve objective
- Must be alternative courses of action from which to choose
- Objectives and constraints must be expressible as linear equations or inequalities

An example

A company produces drugs I and II using machines M1 and M2. One ton of drug I requires 1 hour of processing on machine M1 and 2 hours of processing on machine M2. One ton of drug II requires 3 hours of processing on machine M1 and 1 hour of processing on machine M2. Available times are 9 hours of processing on M1 and 8 hours on M2. Each ton of drug produced (of either type) yields a profit of 1 million rupees. To maximize its profit, how much of each drug should the company produce?

Linear Programming set up:

Let x_1 be the number of tons of drug I produced, and x_2 be the number of tons of drug II produced. The problem is therefore:

Maximize	$x_1 + x_2$	(Profit)	-	Objective Function
Subject to	$x_1 + 3x_2 \leq 9$	(Processing by machine M1)	-	Constraint
	$2x_1 + x_2 \leq 8$	(Processing by machine M2)	-	Constraint
	$x_1, x_2 \geq 0$	(Non negative values)	-	Constraint

Here x_1 and x_2 are the decision variables and to get maximum profit from the available resources it is desirable to obtain the maximum value for the objective function $x_1 + x_2$ retaining the conditions (constraints).

A general Feed Formulation problem:

b_i - required number of units of nutrient i , $i = 1, \dots, m$.

a_{ij} - number of units of nutrient i in one unit of food j , $i = 1, \dots, m$, $j = 1, \dots, n$.

c_j - cost per unit of food j .

x_j - number of units of food j in the diet.

We seek to minimize $c_1x_1 + \dots + c_nx_n$
 subject to $a_{i1}x_1 + \dots + a_{in}x_n \geq b_i$ for all i ,
 $x_1, \dots, x_n \geq 0$.

In matrix notation, we can write this as

$$\min_{\mathbf{x}} \mathbf{c}^T \mathbf{x} \quad \text{subject to} \quad \mathbf{Ax} \geq \mathbf{b}$$

$$\mathbf{x} \geq \mathbf{0}$$

Define the following:

For a Linear Programming solution for feed formulation, information necessary are (i) a list of ingredients that are available for use in the feed and their cost. (ii) The nutrient contents for each of the ingredients. (iii) The nutritional requirements of the species in terms of minimum, maximum or exact quantities needed, and (iv) any physical or non-nutritive limitation which might be imposed because of ingredient characteristics such as limitation of supply, effects on feed mixture, toxic factors, ability of feed to be pelleted etc. There are certain attributes such as palatability or acceptability on which it is difficult to place a numerical value.

No	Ingredients	Protein (a_{ij})	Lipid	NFE	Fibre	DE	Met
1	Fish meal	55.60 a_{11}	12.00	8.10	3.00	3566	1.67
2	Shrimp meal	35.50 a_{12}	7.00	16.00	12.70	2549	0.71
3	Clam meal	55.20 a_{13}	9.70	23.90	4.20	3839	1.96
4	Blood meal	88.50 a_{14}	1.20	3.90	0.40	3974	0.95
5	Poultry byproduct meal	59.90 a_{15}	17.10	5.40	2.10	4076	1.13
6	Hydrolysed feather meal	91.40 a_{16}	3.90	0.50	0.40	4212	0.59

No	Nutrients / Ingredient	Requirements	(b_i)
1	Protein	$\geq 47.8\%$	b_1
2	Lipid	$\leq 14\%$	b_2
3	NFE	$\leq 20\%$	b_3
4	Fibre	$\leq 6\%$	b_4
5	Digestible Energy	≥ 3400 Kcal/Kg	b_5
6	Met	$\geq 0.97\%$	b_6

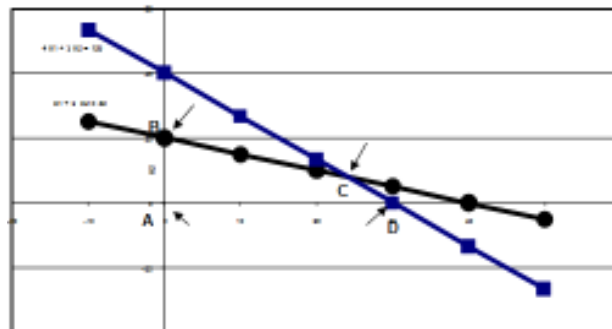
Optimal Solution: Corner Point Method - Graphical method of solving LP (for 2 variables)

The company producing two types of hats

	Hours per unit		
	Types of hat		
Machine	Type-1	Type-2	Capacity
	(X1)	(X2)	(Hrs/Week)
Machine-1	1 hr	2 hr	40 hr
Machine-2	4 hr	3 hr	120 hr
Profit per hat	Rs.4	Rs. 5	

TProblem setup is

$$\begin{aligned} \text{Maximize } Z &= 4X_1 + 5X_2 \\ \text{Subject to: } &1X_1 + 2X_2 \leq 40 \\ &4X_1 + 3X_2 \leq 120 \\ &X_1, X_2 \geq 0 \end{aligned}$$



The constraints represent 4 planes defined by their respective linear equations

$$1 X_1 + 2 X_2 = 40; 4 X_1 + 3 X_2 = 120; \quad X_1 = 0; \quad X_2 = 0$$

These planes intersect and the common area (plane of intersection) will have four vertices shown by the arrows in the above graph. These four points, denoted by A, B, C and D, co-ordinates of the points and value of the objective (profit) function are given in the following table.

$$\text{Profit: } Z = 4X_1 + 5X_2$$

Point	Co-ordinates	Z = 4X ₁ + 5X ₂	Profit
A	(0, 0)	(4)(0) + (5)(0)	0
B	(0, 20)	(4)(0) + (5)(20)	100
C	(24, 8)	(4)(24) + (5)(8)	136
D	(30, 0)	(4)(30) + (5)(0)	120

The vertex that corresponds to maximum value of the objective function is the solution of the linear programming problem. Hence, the solution of the LP is X₁ = 24 and X₂ = 8. So the company will have maximum profit when they produce 24 units of Type-1 hats and 8 units of Type-II hats and the maximum profit is 136 Rupees.

When there are more than two variables in the LP problem, as in most of the cases, the graphical procedure will not be useful. Iterative mathematical procedures (optimization algorithms) are then used to solve the LP problem. The most popular method, which is widely used, is the Simplex method. A recent development in solving general optimization problems is using genetic algorithms which is a method developed based on the theory of evolution in biology. This is a principle borrowed into mathematics from biological science and it involves principles of selection, crossing over, mutation etc. There are different computer software available in market specifically for LP. The famous mathematical software MATLAB has a **linpro** module for solution of LP. The SOLVER add-in of Excel in MS Office is also one software for solving LP.

Linear Programming Example - How to use SOLVER in Excel

Machine	Hours per unit		Capacity
	Type-1	Type-2	
	(X ₁)	(X ₂)	(Hrs/Week)
Machine-1	1 hr	2 hr	40 hr
Machine-2	4 hr	3 hr	120 hr
Profit per hat	Rs.4	Rs. 5	

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Problem setup is

$$\text{Maximize } Z = 4X_1 + 5X_2$$

Subject to:

$$1 X_1 + 2 X_2 \leq 40$$

$$4 X_1 + 3 X_2 \leq 120$$

$$X_1, X_2 \geq 0$$

Developing necessary equations for solving the LP in Excel's Solver:

$$\text{Objective Function: } Z = 4X_1 + 5X_2$$

Data Preparation:

Variables:	X1	X2	
Coefficients:	4	5	array1 is C21:D21
Solution:	1	1	array2 is C22:D22 (Initially some arbitrary value)
Value	9		Determined using the sumproduct function as =SUMPRODUCT(C21:D21,C22:D22) This is the target cell (C23 - to be maximized)

Left hand side of the constraints

Coefficients	X1	X2	Value = Sumproduct(array1,array2)
Equation-1	1	2	3 =SUMPRODUCT(C32:D32,C22:D22)
Equation-2	4	3	7 =SUMPRODUCT(C33:D33,C22:D22)

Right hand side of the constraints

	RHS
Equation-1	40
Equation-2	120

Solver execution:

- Now select solver from the Tools menu of excel after preparation of data.
- Then the window for feeding solver parameters will appear. In the set target cell box click and select the target cell where the value of the objective function for a given solution of variables is given (cell address: C23).
- Choose the maximization option in the radio button options
- In the by changing cells box enter the range where an arbitrary solution for the variables is entered (cell array: C22..C23)
- Press the Add button and you will get a window box to enter the constraints equations.
- For each of the constraints the value calculated using the *sumproduct* function is to be given as the left hand side of the constraint equation is the cell reference.
- Choose the correct sign (<= or >= or =) at the middle and the RHS value reference in the constraint cell and then press add button.
- This is to be repeated for each constraint equation.

- After finishing this close the window box to come back to the window for solver parameters.
- Click the options button to get another window, solver options window.
- Tick the assume non-negative checkbox and press OK button to go back to solver parameters window.
- Now press the solve button to carry out the calculations.
- Once it is finished, the Solver Results window appears where in the Reports portion there is an Answer option. Double click on this to add the results to your workbook as a separate worksheet.
- Final values of the variables as well as the objective function will be available in this worksheet.