A THEORETICAL APPROACH TO THE ATTENUATION COEFFICIENT OF LIGHT IN SEA WATER

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ABSTRACT

The attenuation coefficient of sea water is derived from a formula which is more general than the Lambert's equation. The mean attenuation coefficient which the Secchi disc observations give is obtained therefrom. The constant of the Secchi disc is discussed in the light of the relationship developed. The theoretical concept developed here, falls in accordance with the expressions based on field data.

INTRODUCTION

The attenuation coefficient of light in sea water is an important parameter not only from its physical point of view but also from the view point of its relationship to primary production (Riley, 1956 and Strickland, 1958). The attenuation coefficient of light depends upon absorption and scattering in the medium. The absorption and scattering caused by sea water at different wavelengths of light have been discussed in detail by Jerlov (1967).

Although modern electronic devices (Gall, 1949; Jerlov, 1951 and Sasaki, 1964) are available to determine quite accurately the attenuation coefficients, the Secchi disc is still the simplest device to determine quite easily the mean attenuation coefficient of sea water for visible radiation. Though the device is simple, a theoretical approach towards its perfection makes it a more practical object for use. Tyler (1968) developed a fine theory on Secchi disc. His aim was, however, to divide the attenuation coefficient as determined by Secchi disc into its primary components, namely, the two coefficients for collimated and diffused light.

Certain suggestions have appeared in modern literature on the use of Secchi disc (Jones and Wills, 1956; Hanaoka, 1957 and Qasim, Bhattathiri and Abidi, 1968) with some modifications in the expression originally determined by Poole and Atkins (1929) based on their studies in the English Channel. These modifications are based on empirical relations developed between the Secchi depths and attenuation coefficients, determining the latter with the help of a photometer.

In the present communication, the author makes an attempt to explain the 'constant' of Secchi disc purely on a theoretical basis.
The intensity of a parallel-beam radiation of a particular wavelength decreases along its path in pure water, obeying the Bouguer Law (also called the Lambert Law). In the following discussion the absorption of light includes scattering as well. The absorption of radiation in sea water is a complicated phenomenon because of the suspended particles and dissolved substances (Kalle, 1966). Nevertheless, the general equation of absorption of radiation in sea water may be written as

\[ I_z = I_o e^{-\alpha Z^{\beta}} \]  

where \( I_z \) is the intensity of radiation at depth \( Z \), \( I_o \) is the intensity at the sea surface \( (Z = 0) \) and \( \alpha \) and \( \beta \) are positive constants. The constant \( \alpha \) is characteristic of the 'type' of sea water.

The particular value of \( \beta \) as unity refers to Lambert Law. Further, by making use of the data given by Sverdrup (1945) on the percentages of incident energy per unit area at different levels in the sea, one obtains that \( \beta = \frac{1}{2} \) and \( \alpha \) attains values 0.371, 0.545, 0.862, 1.562 and 1.871 characterising the five types of sea water, namely, pure sea water, clearest oceanic water, average oceanic water, average coastal water and turbid coastal water respectively. It is noteworthy that the values of \( \alpha \) characteristically increase in the order of the type of sea water ranging from pure sea water to the coastal turbid water.

Taking logarithms on both sides of eq. (1),

\[ \ln I_z = \ln I_o - \alpha Z^{\beta} \]  

differentiating the eq. (2) with respect to \( z \),

\[ \frac{d}{dz} \left( \ln I_z \right) = -\alpha \beta Z^{\beta-1} \]

The attenuation coefficient, \( K \), of sea water may be defined as \( -\lim (\ln I/\delta Z) \) as \( \delta Z \) tends to become zero. Therefore from the eq. (3),

\[ K = \alpha \beta Z^{\beta-1} \]

Since \( K \) is regarded as positive and it would decrease with depth (Sverdrup, 1945), it is clear from the above equation that \( \beta \) should take a value greater than zero and less than unity \( (0 < \beta < 1) \). If so, the attenuation coefficient \( K \) will be a nonlinear function of depth, the power to which the depth is raised being a number between 0 and -1. It is also clear from the eq. (4) that the vertical profile of attenuation coefficient would, by virtue of \( \alpha \), depend upon the type of sea water. Hence the attenuation coefficient is a useful means for identifying the types of sea water while making optical observations in the sea.
For the Secchi disc, let \( D \) be the vertical depth (in metres) at which the disc just 'disappears' to an observer on board a ship. Let \( I_{BO} \) be the intensity of radiation reaching the sea surface from the disc. \( I_{BO} \) is a negligibly small quantity. Thus the radiation beam (visible light) incident at the sea surface with intensity \( I_0 \) suffers absorption as it traverses the vertical path from the sea surface to the disc and then back from the disc to the sea surface, diminishing finally to a negligible amount \( I_{BO} \).

As defined by eq. (4) the attenuation coefficient \( K_z \) at depth \( z \) is given by

\[
K_z = \alpha \exp(-z) \quad \text{for} \quad \alpha > 0 \quad \text{and} \quad z > 0
\]

Hence, \( \bar{K} \) the mean attenuation coefficient for the depth interval \( D \), from the surface of the sea to the depth at which the Secchi disc just disappears, is obtained by

\[
\bar{K} = \frac{\int_0^D K_z \, dz}{\int_0^D dz} = \alpha D^\alpha - 1
\]

According to the eq. (1) the intensity of radiation \( I_D \) at the disc at depth \( D \) is given by

\[
I_D = I_0 \exp(-\alpha D)
\]

and also

\[
I_{BO} = I_D \exp(-\alpha D)
\]

Eliminating \( I_D \) from eq. (6) and (7),

\[
-I_{BO} = I_0 \exp(-2\alpha D)
\]

or

\[
-I_{BO}/I_0 = e^{-2\alpha D}
\]

Let the ratio \( I_{BO}/I_0 \) be denoted by \( n/100 \) where \( n \) is a small positive number. Then eq. (8) becomes

\[
-2\alpha \exp(-D) = n/100
\]

or

\[
\alpha = \frac{-2 \log(n/100)}{2D}
\]

where the logarithm is taken to the base 10. Substituting the above value of \( \alpha \) in eq. (5) for the mean attenuation coefficient, we obtain

\[
\bar{K} = \frac{2.303\log\left(\frac{n}{100}\right)}{2D}
\]
Assuming that there is no loss of light due to absorption at the disc and neglecting the differences in the eye power of individual observers, the percentage loss of light \( n \) may be attributed to the type of sea water. This is because the intensity of light coming from the Secchi disc is judged against the background illumination of the surrounding waters. This is evident from eq. (9). Strickland (1958) has stated that the general background illumination at the sea surface is of the order of 2 to 3 percent of the vertical intensity of the incident light at the sea surface under natural conditions. A similar expression was derived by him on the assumption of the validity of Lambert Law. Further, if \( p \) becomes unity, no distinction could be made between \( \alpha \) and \( K \), as they become equal to one another.

Let us give different values to \( n \) in eq. (10)

\[
\begin{align*}
\text{for } n = 1, & \quad K = \frac{2.3}{D}, \\
\text{for } n = 2, & \quad K = \frac{1.9}{D}, \\
\text{for } n = 3, & \quad K = \frac{1.7}{D}, \\
\text{for } n = 4, & \quad K = \frac{1.6}{D}, \\
\text{for } n = 5, & \quad K = \frac{1.5}{D}
\end{align*}
\]  

(11)

and so on. It may be interesting to view the 'constant' of the Secchi disc in the light of the above discussion. Since \( \alpha \) is a function of \( n \) (eq. 9), a variation in \( n \) (eq. 10) may also mean a variation in \( \alpha \) which means a change in water type (optically). It is of theoretical interest that the constant of the Secchi disc is 1.9 (eq. 11) for waters requiring 2% \((n = 2)\) back illumination at the sea surface. For waters requiring 3% \((n = 3)\), the constant is 1.7 and for those requiring 5% \((n = 5)\), it is 1.5 only.

It may also be mentioned here that Poole and Atkins (1929) obtained a constant of 1.7 by photoelectric measurements for waters of the English Channel. Jones and Wills (1956) determined the attenuation coefficient with a transparency meter at various depths and also took the Secchi disc observations in the waters around Plymouth and the mouth of the Thames and thereby arrived at an empirical relation between the Secchi depth and the attenuation coefficient. The relationship yielded a constant of 1.9, if very turbid waters were excluded. (See also Strickland, 1958). Qasim et al. (1968) obtained a value of 1.5 for the very turbid estuarine waters at Cochin. Their observations were based on data collected in two ways (i) transmission of illumination at a depth where the disc disappears and (ii) upward scattered light reaching the observer. They used a lux-meter for comparison. It may be concluded that the theoretical concept developed here, falls in accordance with the expressions based on field data.

\section*{References}


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