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Tree Network-balanced Designs for Agroforestry Trials

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SUMMARY

This study was carried out to develop designs for agroforestry experiments involving multiple trees and multiple crop species. A linear network effects model incorporating tree effects from adjacent plots has been considered, and a general method of constructing a class of designs balanced for tree network effects has been developed. The proposed designs are partially variance balanced for estimating direct effects of tree-crop combinations, with the tree-crop combinations following a two-associate class Group Divisible (GD) association scheme. It has been shown that these designs are highly efficient. Also, the layout in separate arrays permits each replication of these designs to be suitably adapted in a different location. Hence, the designs have promising application potential in agroforestry experimentations involving multiple trees and multiple crop species with resource limitations.

Keywords: Adjacency matrix, Canonical efficiency factor, Group divisible association scheme, Partially variance balanced.

1. INTRODUCTION

The use of appropriate designs is very important in agroforestry experimentation. In experimental setup, the arrangements and interactions of tree and crop components within/across plots deserve important consideration. The type and number of components involved in a particular agroforestry trial is dependent on the purpose of the trial. Whatever the purpose of the experiment may be, steps must be taken to ensure that the design used provides more precise estimates of effects of interest through proper accountability of sources of variation.

Generally, experimental material in agroforestry is more heterogeneous because multiple components are involved. Whenever the heterogeneity in experimental material is believed to be traceable to two sources, the experimental plots may be arranged in row and column arrays giving rise to row-column designs. Designs with row-column structure have a place in practical experimentation and have received a reasonable amount of research attention. Freeman (1979) gave methods of constructing row-column designs balanced for neighbours with and without border plots. Federer and Basford (1991) presented three methods of constructing balanced nearest-neighbour row-column designs using models with two-dimensional layouts. An algorithmic approach has also been used to generate some neighbour balanced row-column designs (Chan and Eccleston, 2003). Varghese *et al.* (2011, 2014) developed methods of constructing neighbour balanced row-column designs accounting for neighbour effects from the four adjacent neighbours and studied the characterization properties of those designs. Freeman and Williams (2016) discussed row and column designs with special attention to those in which the intersection of any row and any column results in only one experimental unit or plot.

Sometimes it may be required to layout an agroforestry experiment in multiple locations due to scarcity or heterogeneity of land, requirements of uniform land preparation, and cultural practices. Such a situation requires the use of resolvable designs in

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which every sub-location will contain each treatment an equal number of times. Williams *et al.* (2006) studied efficient resolvable spatial row-column designs under a two-dimensional linear variance model. Piepho *et al.* (2015) carried out a brief review of row-column designs and showed that when such designs are resolvable they allow for even distribution of treatments among complete replicates.

Most of the previous investigations have considered models under block or row-column setups with directional neighbour effects. Though such models are appropriate for most agricultural trials, they may become inadequate when applied to agroforestry experiments due to the growth characteristics of trees. Trees constitute a major component and their influences could be experienced in all directions beyond the plots on which they stand, thus constituting neighbour or spill over effects. Moreover, interactions among trees on different plots can be seen as network of connectivity among the plots. Therefore the problem of developing designs for agroforestry trials gets much complicated when multiple trees and multiple crop species are being considered. When neighbour effects (from trees) are assumed to be same irrespective of the direction of the neighbours, a linear network effects model can be appropriately used. Parker *et al.* (2017) studied the linear network effects model in optimal designs on connected units with application to social networks. Unfortunately, little is known about any specific agroforestry design which can cater for all the above scenarios, hence the need to develop such designs.

In this study, we attempt to develop designs that are balanced for tree network effects pertaining to agroforestry experiments involving multi-tree and multi-crop species. Such designs will yield more precise estimates of estimated variance pertaining to tree-crop combinations after eliminating tree network effects, and permit for better understanding of the relationships among the different components of an agroforestry system.

2. CONSTRUCTION AND ANALYSIS OF TNetBD

Network is a concept drawn from Graph theory. According to Chartrand and Zhang (2012), a graph G refers to a finite nonempty set of N vertices, which are connected by a set of lines called edges (£). Usually, the interactive structure among individuals or experimental units (EUs) can easily be specified when the EUs are considered as nodes or vertices, and the relationships or connections among them as edges in a graph. In designing of experiments, a network refers to an undirected graph comprising a collection of $|N| = n$ EUs on which some treatment is applied, and edges $£ \subseteq (N \times N)$ which connect the EUs together. The connections between adjacent units are specified by an $n \times n$ adjacency matrix $\boldsymbol{A} = \{a_{id}\}$, where $a_{id} = 1$ if the i^{th} and d^{th} EUs are directly connected and $a_{id} = 0$ otherwise $(i, d = 1, 2, \ldots, n)$. Usually, $\boldsymbol{A}$ is symmetric and $a_{ii} = 0$ for all $i = d$. The adjacency matrix $\boldsymbol{A}$ is very important because it specifies the connectivity among the EUs which is the fundamental assumption in linear network effects model. We illustrate this by considering a field divided into plots of which 12 are regular (Fig. 1). The adjacency matrix of the 12 regular plots in the layout is given below.

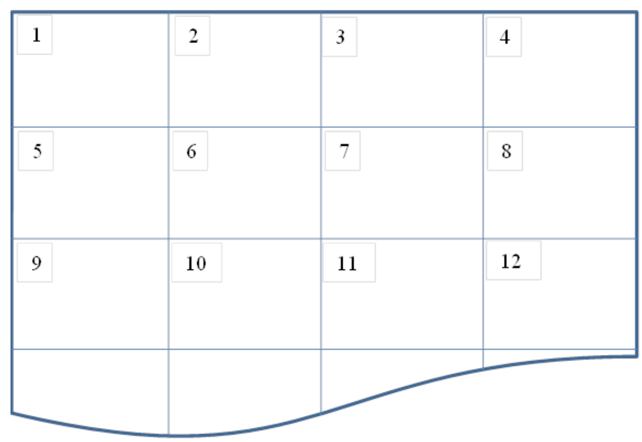

Fig. 1. Layout of plots in a field

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

We adapt the network concept for use in agroforestry experimentation by considering that an agroforestry field, G, is partitioned into m mutually exclusive square plots. The partition is such that there are n inner plots and $m-n$ edge plots (serving as borders). Therefore, each inner plot will have four adjacent plots connected directly to it.

2.1 Experimental setup and model

Consider an agroforestry experiment in which multiple (different) tree species and multiple crop species/varieties are planted on n plots, with each plot receiving only one tree species and one crop species. Let there be t tree species and c crop species such that together they form a total of tc tree-crop combinations. We state here that, for the class of designs under consideration, the construction is suitable and characterization properties are easily obtained when equal numbers of tree and crop species are used. Thus, $t = c = v$ say, and $tc = v^2$ hence forth. We denote by Y_i the response of "direct effect" ($\tau_{j,i}$) of tree-crop combination from i^{th} plot $(i = 1, 2, \ldots, n)$ having j^{th} tree-crop combination $(j = 1, 2, \ldots, v^2)$, and "tree network effect" ($\delta_{l,k}$) from k^{th} plot $(k = 1, 2, \ldots, n)$ of the four adjacent plots on left, right, top and bottom of plot i, where tree-crop combination l is planted ($l = 1, 2, \ldots v^2$). Note that tree species on plot i is different from tree species on plot k, thus no self-neighbours. Assuming that all observations are uncorrelated and have common variance, then the general form of the linear network effects model could be written as:

$$E(\boldsymbol{Y}) = \mu\mathbf{1} + \boldsymbol{F\tau} + \boldsymbol{N\delta} = \boldsymbol{X\theta} \tag{2.1}$$

where, $E()$ refers to the expected value of observation, $\boldsymbol{Y}$ is an n-vector of observations measured from n plots, μ is grand mean, $\mathbf{1}$ is an n-vector of unities, $\boldsymbol{F}$ is an $n \times v^2$ design matrix of observations versus direct tree-crop combinations, $\boldsymbol{\tau}$ is a v^2-vector of direct effects of tree-crop combinations, $\boldsymbol{N} = \boldsymbol{AT}$ is an $n \times v$ design matrix of observations versus all (non-directional) adjacent trees, $\boldsymbol{A}$ is an $n \times n$ matrix of adjacency of the plots, $\boldsymbol{T}$ is an $n \times v$ design matrix of observations versus tree species, $\boldsymbol{\delta}$ is a v-vector of all network (adjacency) effects, $\boldsymbol{X} = [\mathbf{1} \quad \boldsymbol{F} \quad \boldsymbol{N}]$ is a $n \times (v^2 + v + 1)$ design matrix which we partitioned as $\boldsymbol{X}_1 = [\boldsymbol{F} \quad \boldsymbol{N}]$ and $\boldsymbol{x}_2 = \mathbf{1}$, and $\boldsymbol{\theta}' = [\mu \quad \boldsymbol{\tau}' \quad \boldsymbol{\delta}']$ is a $(v^2 + v + 1)$-vector of parameters. Here, $\boldsymbol{F} = [\boldsymbol{f}_1 \quad \boldsymbol{f}_2 \quad \cdots \quad \boldsymbol{f}_j \quad \cdots \quad \boldsymbol{f}_{v^2}]$ where $\boldsymbol{f}_j$ is an indicator vector defined as:

$$\boldsymbol{f}_j = \begin{cases} 1, \textit{if } \ j^{\text{th}} \text{ tree} - \text{crop combination is planted on plot } i \\ 0, \ \text{otherwise}. \end{cases}$$

Let us rearrange the components of $\boldsymbol{X}$ as $\boldsymbol{X} = [\boldsymbol{X}_1 \quad \boldsymbol{x}_2] = [\boldsymbol{F} \quad \boldsymbol{N} \quad \mathbf{1}]$ so that $\boldsymbol{X}'\boldsymbol{X} = \boldsymbol{Z}$ where, $\boldsymbol{Z}$ is known as the Fisher's information matrix and has the following structure,

$$\boldsymbol{Z} = \begin{bmatrix} \boldsymbol{R}_1 & \boldsymbol{H} & \boldsymbol{r}_1 \\ \boldsymbol{H}' & \boldsymbol{R}_2 & \boldsymbol{r}_2 \\ \boldsymbol{r}_1' & \boldsymbol{r}_2' & n \end{bmatrix} \tag{2.2}$$

where, $\boldsymbol{R}_1 = \boldsymbol{F}'\boldsymbol{F}$ is a $v^2 \times v^2$ diagonal matrix of replications of tree-crop combinations, $\boldsymbol{R}_2 = \boldsymbol{N}'\boldsymbol{N}$ is a $v \times v$ matrix of cross products of concurrence of each tree species with other tree species, $\boldsymbol{H} = \boldsymbol{F}'\boldsymbol{N}$ is a $v^2 \times v$ matrix of concurrence of tree-crop combinations with each tree species, $\boldsymbol{r}_1 = \boldsymbol{F}'\mathbf{1}$ is a v^2-vector of replications of tree-crop combinations, $\boldsymbol{r}_2 = \boldsymbol{N}'\mathbf{1}$ is a v-vector of replications of all neighbouring trees, and $n = \mathbf{1}_n'\mathbf{1}_n$ is a scalar which represents the total number of plots or EUs on which tree-crop combinations are planted and from which measurements are taken.

Now we define some terms associated with network designs for agroforestry trials, specifically taking into account the proposed class of designs. Let there be m plots arranged in g row-column arrays such that in each array, there are n inner plots and $m-n$ border plots. The number of inner plots in a row is the row length, the number of inner plots in a column is the column length and let row length $= \alpha$ and column length $= \beta$. The $n(= \alpha\beta)$ plots are planted with v^2 tree-crop combinations. We denote by $i(\mathscr{b}, \mathscr{k})$ the tree species assigned to i^{th} plot in the row ($\mathscr{b}$) and column ($\mathscr{k}$), where $\mathscr{b} = 0, 1, 2, \ldots, \beta, \beta + 1$; $\mathscr{k} = 0, 1, 2, \ldots, \alpha, \alpha + 1$. Here, $0, \beta + 1$ represent border rows and $0, \alpha + 1$ represent border columns.

Definition 1: The influence that tree species $i(\mathscr{b}, \mathscr{k})$ exerts on adjacent plots is referred to as *tree network effects*.

Definition 2: A design for multiple trees and multiple crops species arranged in rows and columns is referred to as *design for multi-tree multi-crop species* (D-mTC) if it is possible to group the entire rows and columns of the design into g arrays wherein each array consists of each tree-crop combination exactly once. A D-mTC is said to be network balanced (TNetBD-mTC) for tree species if every tree-crop combination has every other tree species (except the species occurring in the combination) appearing as neighbour a constant (u) number of times.

Definition 3: A TNetBD-mTC is said to be *circular* if $i(b,0)=i(b,\alpha)$, $i(b,1)=i(b,\alpha+1)$, $i(0,k)=i(\beta,k)$ and $i(1,k)=i(\beta+1,k)$.

Definition 4: *Association Scheme*: In a network balanced design for multi-tree multi-crops species with $tc=v^2$ tree-crop combinations, any two tree-crop combinations tc and $t'c'$ are said to be first associates if they have the same tree species $(t=t')$ and different crop species $(c\neq c')$, and second associates otherwise. Thus, all tree-crop combinations involving different trees with same crop as well as different trees with different crops are second associates.

Definition 5: A TNetBD-mTC is said to be *partially variance balanced* if the contrasts pertaining to different sets of tree-crop combinations are estimated with different variances, but within same set with a constant variance. For the designs under consideration, contrasts pertaining to tree-crop combinations having same tree but different crops are estimated with the same variance (say, V_1), and the contrasts pertaining to all other tree-crop combinations are estimated with same variance (say, V_2). From now onwards, we shall be using TNetBD in place of TNetBD-mTC since the designs are entirely for multi-tree and multi-crop trials.

2.2 Method of construction

A TNetBD can be constructed by following the steps outlined below. Let there be v tree species denoted by (0, 1, 2, …, $v-1$) and v crop species denoted by ($a, b, c, \ldots$, etc), where $v(\geq 5)$ is a prime number.

Step 1: Starting with '0', develop the g^{th} array by making increment of $g\left(g=1,2,\ldots,\frac{(v-1)}{2}\right)$ to row and column elements cyclically (mod v). This produces $(v-1)/2$ square arrays each of size $v\times v$.

Step 2: In each array, allocate the symbols $(a, b, c, \ldots,$ etc) in such a way that symbol a occurs with all the elements of row 1, b with all the elements of row 2, c with all the elements of row 3 and so on.

Step 3: Make each array circular in all directions by providing appropriate border plots with the levels of the v tree species. The border plots act as guard plots and contribute in tree network balance but observations are not taken from them.

We illustrate the method of construction with an example below.

Example 1: Consider an experiment with $v=5$ tree species and $c=5$ crop species. The 2 initial square arrays are developed as:

Array I				
0	1	2	3	4
1	2	3	4	0
2	3	4	0	1
3	4	0	1	2
4	0	1	2	3

Array II				
0	2	4	1	3
2	4	1	3	0
4	1	3	0	2
1	3	0	2	4
3	0	2	4	1

The final design of tree-crop combinations is given below.

	4	0	1	2	3	
4	0a	1a	2a	3a	4a	0
0	1b	2b	3b	4b	0b	1
1	2c	3c	4c	0c	1c	2
2	3d	4d	0d	1d	2d	3
3	4e	0e	1e	2e	3e	4
	0	1	2	3	4	

	3	0	2	4	1	
3	0a	2a	4a	1a	3a	0
0	2b	4b	1b	3b	0b	2
2	4c	1c	3c	0c	2c	4
4	1d	3d	0d	2d	4d	1
1	3e	0e	2e	4e	1e	3
	0	2	4	1	3	

2.3 Principles of experimentation

To ensure validity and sensitivity of the analysis of data from the proposed designs, the designs must be subjected to the three basic principles of experimental design namely, replication, randomization and local control.

In the present study, each tree-crop combination is replicated $(v-1)/2$ times. If more replication is required, a design with $(v-1)$ arrays should be used in which case tree-crop combination will be replicated $(v-1)$ times.

The proposed designs can be randomized through a restricted randomization procedure, which consists of the following steps:

Step 1. Randomly allot $0,1,2,\ldots,(v-1)$ to the v tree species.

Step 2. Choose any $(v-1)/2$ consecutive arrays randomly from the total possible $(v-1)$ arrays.

Step 3. Randomize the chosen $(v-1)/2$ arrays.

Step 4. Randomly allocate the v crop symbols $(a, b, c, \ldots,$ *etc*.) to rows in each array.

Step 5. Within each array, apply circular randomization of either rows or columns but not both at the same time.

Step 1 ensures that each tree species has equal chance of being allotted to any plot. Step 5 gives all tree species equal chance to be allotted the first plot in row (column) but once first allotment is done, rest of the tree species are allotted in a circular order in the remaining rows (columns). This means that every TNetBD has v different arrangements of rows (columns) upon randomization. For instance, the first row of the first array in Example 1 above is $4\ |0a\ \ 1a\ \ 2a\ \ 3a\ \ 4a|\ 0$, and upon randomizing columns the final arrangement in the first row may become $2\ |3a\ \ 4a\ \ 0a\ \ 1a\ \ 2a|\ 3$. If one chooses to randomize rows instead, the first column of the first array may become, say, $1\ |2c\ \ 3d\ \ 4e\ \ 0a\ \ 1b|\ 2$.

The third principle - local control - is relaxed in TNetBD. We assume a network balanced model where each tree-crop combination is combinatorially balanced for tree species and network effects of trees from adjacent plots.

Remark: When constructing the designs, it is advisable to finish the randomization process before taking appropriate borders to make the final layout circular.

2.4 Analysis of variance using synthetic data

Under Model (2.1), the total variation can be partitioned as variation due to direct effects of tree-crop combinations, variation due to tree network effects and the random error component. The design given in Example 1 is now shown below with synthetic data for the purpose of illustration. We consider five tree species as Siris (0), Neem (1), Shisham (2), Babul (3) and Ghaf (4) and five crop species as Barley (a), Gram (b), Wheat (c), Moong (d) and Maize (e). The data values are in units of 1000 rupees representing monetary values of yields. In each cell, the first value in the parenthesis is for fodder yield of tree while the second value is for crop yield.

Array I

0a (1.895, 60.9)	**1a** (1.51, 67.91)	**2a** (0.784, 76.62)	**3a** (1.19, 72.17)	**4a** (1.91, 65.49)
1b (1.476, 62.72)	**2b** (0.854, 65.84)	**3b** (1.28, 60.33)	**4b** (2.082, 74.56)	**0b** (1.97, 68.43)
2c (0.89, 79.91)	**3c** (1.24, 77.01)	**4c** (1.84, 68.63)	**0c** (1.72, 66.99)	**1c** (1.58, 79.8)
3d (1.175, 78.98)	**4d** (2.024, 74.64)	**0d** (1.78, 68.72)	**1d** (1.25, 72.71)	**2d** (0.808, 69.86)
4e (1.94, 62.38)	**0e** (1.74, 63.99)	**1e** (1.47, 64.7)	**2e** (0.355, 60.72)	**3e** (1.296, 72.55)

Array II

0a (1.734, 62.78)	**2a** (0.975, 71.15)	**4a** (2.09, 78.48)	**1a** (1.528, 74.38)	**3a** (1.375, 62.16)
2b (0.73, 66.88)	**4b** (1.962, 62.96)	**1b** (1.57, 63.24)	**3b** (1.33, 75.69)	**0b** (1.712, 66.91)
4c (1.98, 78.87)	**1c** (1.575, 73.92)	**3c** (1.13, 67.27)	**0c** (1.868, 65.47)	**2c** (0.76, 77.15)
1d (1.12, 79.71)	**3d** (1.234, 77.94)	**0d** (1.695, 66.82)	**2d** (0.737, 74.46)	**4d** (1.92, 71.56)
3e (1.295, 60.44)	**0e** (1.65, 66.24)	**2e** (0.54, 66.95)	**4e** (2.095, 62.57)	**1e** (1.416,74.72)

To analyse the data, we use the total (tree + crop) monetary value as the response from each plot. Each plot response is influenced twice by each of the other tree species surrounding the plot, hence for analysis purpose each value has been considered twice. The SAS code for the analysis is given in Appendix I. The combined ANOVA for direct tree-crop adjusted (network unadjusted) and direct tree-crop unadjusted (network adjusted) is given in Table 1.

Agricultural researchers are mainly interested in comparing between tree species as well as crop varieties within each tree species. For this purpose, the variation due to direct effects of tree-crop combinations can be further bifurcated into variation due to tree species and crop species within tree species with 4 and 20 degrees of freedom respectively. Accordingly, the orthogonally partitioned sums of squares (SS) are obtained and given in the ANOVA table (Table 1).

Table 1. ANOVA for synthetic tree-crop data of TNetBD for $v = 5$

Source of variation		DF	Type I SS	Mean Square	F value	Pr> F
Tree-crop combination (adj)		24	2274.246285	94.760262	5.17	<0.0001
	Tree species	4	372.747983	93.186996	5.09	0.0012
	Crop(Tree)	20	1901.498302	95.074915	5.19	<0.0001
Tree network (unadj)		4	22.497341	5.624335	0.31	0.8723
OR						
Tree-crop combination (unadj)		24	2250.500399	93.770850	5.12	<0.0001
	Tree species	4	349.002097	87.250524	4.76	0.0018
	Crop(Tree)	20	1901.498302	95.074915	5.19	<0.0001
Tree network (adj)		4	46.243227	11.560807	0.63	0.6418
Error		71	1300.207333	18.312779		
Total		99	3596.950959			

DF = degrees of freedom, SS = Sum of squares, Pr = probability

3. PROPERTIES OF TNetBD

Every TNetBD so obtained under Section 2.2 exhibits the following properties. The arrangement of v^2 tree-crop combinations into $(v-1)/2$ circular square arrays (separated by sufficient gaps) results in a combinatorially balanced and equi-replicated network balanced design. Each tree-crop combination is replicated $r_1 = (v-1)/2$ times and has all the other tree species occurring in the adjacent plots $r_2 = 2v(v-1)$ times. The arrangement is such that the v^2 tree-crop combinations are grouped into $(v-1)/2$ arrays with each combination appearing in each array once. Every tree-crop combination has $v-1$ other combinations as first associates and the remaining $v(v-1)$ combinations as second associates. From Equation (2.1), the direct effects of all tree-crop combinations are accounted for by $\boldsymbol{\tau}$ while the effects of trees from the four adjacent plots (including border plots) considered for each main plot, are accounted for by the network effects parameter $\boldsymbol{\delta}$.

3.1 Information matrix

Upon constructing the designs and getting the design matrices $\boldsymbol{F}$, $\boldsymbol{T}$, $\boldsymbol{N}$ and $\boldsymbol{A}$, the sub-matrices and vectors in $\boldsymbol{Z}$ from Equation (2.2) are obtained as:

$$\boldsymbol{R}_1 = \frac{(v-1)}{2}\boldsymbol{I}_{v^2}, \qquad \boldsymbol{H} = -2[\boldsymbol{I}_v - \boldsymbol{J}_v]\otimes\boldsymbol{1}_v,$$

$$\boldsymbol{R}_2 = 4v[(v-2)\boldsymbol{I}_v + \boldsymbol{J}_v], \qquad \boldsymbol{r}_1 = \frac{(v-1)}{2}\boldsymbol{1}_{v^2},$$

$$\boldsymbol{r}_2 = 2v(v-1)\boldsymbol{1}_v, n = \frac{v^2(v-1)}{2},$$

where, $\boldsymbol{I}$ is identity matrix, $\boldsymbol{J}$ is a matrix of unities, $\boldsymbol{1}$ is a vector of unities and $\otimes$ is a symbol for Kronecker product. We know that the normal equations from least squares estimation are generally expressed as $\boldsymbol{X}'\boldsymbol{X}\boldsymbol{\theta} = \boldsymbol{X}'\boldsymbol{Y}$. Since we partitioned $\boldsymbol{X}$ as $\boldsymbol{X} = [\boldsymbol{X}_1 \quad \boldsymbol{x}_2]$ with corresponding parameters as $\boldsymbol{\theta}' = [\boldsymbol{\theta}_1' \quad \theta_2]$ under Equation (2.1), solving the normal equations with respect to parameters of interest $(\boldsymbol{\theta}_1)$ yields $\boldsymbol{C}_\theta\boldsymbol{\theta}_1 = \boldsymbol{Q}$ where,

$\boldsymbol{C}_\theta = \boldsymbol{X}_1'\boldsymbol{X}_1 - \boldsymbol{X}_1'\boldsymbol{x}_2(\boldsymbol{x}_2'\boldsymbol{x}_2)^{-1}\boldsymbol{x}_2'\boldsymbol{X}_1$ is the joint information matrix and

$\boldsymbol{Q} = \boldsymbol{X}_1'\boldsymbol{Y} - \boldsymbol{X}_1'\boldsymbol{x}_2(\boldsymbol{x}_2'\boldsymbol{x}_2)^{-1}\boldsymbol{x}_2'\boldsymbol{Y}$ is the vector of adjusted tree-crop combinations totals.

So the components of $\boldsymbol{Z}$ are used to derive the joint information matrix $\boldsymbol{C}_\theta$ for estimation of direct tree and network effects. Let $\boldsymbol{C}_\theta$ be partitioned into sub-matrices as:

$$\boldsymbol{C}_\theta = \begin{bmatrix} \boldsymbol{C}_{11} & \boldsymbol{C}_{12} \\ \boldsymbol{C}_{21} & \boldsymbol{C}_{22} \end{bmatrix}.$$

The sub-matrices in $\boldsymbol{C}_\theta$ are first obtained in the following steps:

$$\boldsymbol{C}_{11} = \frac{(v-1)}{2}\boldsymbol{I}_{v^2} - \frac{2(v-1)^2}{4v^2(v-1)}\boldsymbol{1}_{v^2}\boldsymbol{1}_{v^2}' = \frac{(v-1)}{2}\left[\boldsymbol{I}_{v^2} - \frac{1}{v^2}\boldsymbol{J}_{v^2}\right].$$

Next,

$$\boldsymbol{C}_{12} = \boldsymbol{C}_{21}' = \left(-2[\boldsymbol{I}_v - \boldsymbol{J}_v]\otimes\boldsymbol{1}_v - \frac{4v(v-1)^2}{2v^2(v-1)}\boldsymbol{1}_{v^2}\boldsymbol{1}_v'\right),$$

note that $\boldsymbol{1}_{v^2} = \boldsymbol{1}_v\otimes\boldsymbol{1}_v$

$$= -2\left[\boldsymbol{I}_v - \frac{1}{v}\boldsymbol{J}_v\right]\otimes\boldsymbol{1}_v$$

and $$\boldsymbol{C}_{22} = \left(4v[(v-2)\boldsymbol{I}_v + \boldsymbol{J}_v] - \frac{2(2v(v-1))^2}{v^2(v-1)}\boldsymbol{1}_v\boldsymbol{1}_v'\right)$$

$$= 4v(v-2)\left[\boldsymbol{I}_v - \frac{1}{v}\boldsymbol{J}_v\right]$$

Writing these sub-matrices together gives the joint information matrix. Thus,

$$\boldsymbol{C}_\theta = \begin{bmatrix} \frac{(v-1)}{2}\left[\boldsymbol{I}_{v^2} - \frac{1}{v^2}\boldsymbol{J}_{v^2}\right] & -2\left[\boldsymbol{I}_v - \frac{1}{v}\boldsymbol{J}_v\right]\otimes\boldsymbol{1}_v \\ -2\left[\boldsymbol{I}_v - \frac{1}{v}\boldsymbol{J}_v\right]\otimes\boldsymbol{1}_v' & 4v(v-2)\left[\boldsymbol{I}_v - \frac{1}{v}\boldsymbol{J}_v\right] \end{bmatrix}. \tag{3.1}$$

We wish to estimate variance of elementary contrasts pertaining to the direct effects of tree-crop combination. This can be achieved through computation of information matrix pertaining to direct effects after eliminating network effects from $\boldsymbol{C}_{\theta}$ given in (3.1).

Thus, $\boldsymbol{C}_{dir} = \boldsymbol{C}_{11} - \boldsymbol{C}_{12}\boldsymbol{C}_{22}^{-}\boldsymbol{C}_{21}$, where $\boldsymbol{C}_{dir}$ is the information matrix for estimation of direct effects of tree-crop combinations and $\boldsymbol{C}_{22}^{-}$ is the generalized inverse of $\boldsymbol{C}_{22}$. It can easily be seen that $\boldsymbol{C}_{22}$ is expressible as a product of a coefficient and an idempotent matrix hence $\boldsymbol{C}_{22}^{-}$ is obtained as:

$$\boldsymbol{C}_{22}^{-} = \frac{1}{4p^2 v(v-2)}\left[\boldsymbol{I}_v - \frac{1}{v}\boldsymbol{J}_v\right].$$

By defining matrix products, $\boldsymbol{I}_v \otimes \boldsymbol{1}_v = \boldsymbol{I}_{v^2 \times v}$, $\boldsymbol{J}_v \otimes \boldsymbol{1}_v = \boldsymbol{J}_{v^2 \times v}$ and $\boldsymbol{I}_{v^2 \times v}\boldsymbol{I}_{v \times v^2} = \boldsymbol{D}_{v^2}$, the following can be obtained in simplified forms:

$$\boldsymbol{C}_{12} = \boldsymbol{C}_{21}' = -2\left[\boldsymbol{I}_{v^2 \times v} - \frac{1}{v}\boldsymbol{J}_{v^2 \times v}\right],$$

$$\boldsymbol{C}_{12}\boldsymbol{C}_{22}^{-} = \frac{-1}{2v(v-2)}\left[\boldsymbol{I}_{v^2 \times v} - \frac{1}{v}\boldsymbol{J}_{v^2 \times v}\right],$$

$$\boldsymbol{C}_{12}\boldsymbol{C}_{22}^{-}\boldsymbol{C}_{21} = \frac{1}{v(v-2)}\left[\boldsymbol{D}_{v^2} - \frac{1}{v}\boldsymbol{J}_{v^2}\right],$$

$$\boldsymbol{C}_{dir} = \frac{(v-1)}{2}\boldsymbol{I}_{v^2} - \frac{1}{v(v-2)}\boldsymbol{D}_{v^2} - \frac{(v-3)}{2v(v-2)}\boldsymbol{J}_{v^2}.$$

Further, let $\boldsymbol{D}_{v^2}$ be expressed as a sum of two matrices, say $\boldsymbol{D}_{v^2} = \boldsymbol{I}_{v^2} + \boldsymbol{E}_{v^2}$, and $\boldsymbol{J}_{v^2}$ be expressed as a sum of three matrices thus, $\boldsymbol{J}_{v^2} = \boldsymbol{I}_{v^2} + \boldsymbol{E}_{v^2} + \boldsymbol{M}_{v^2}$. Therefore the information matrix for estimation of direct effects is obtained as:

$$\boldsymbol{C}_{dir} = \left(\frac{(v-1)(v^2-2v-1)}{2v(v-2)}\right)\boldsymbol{I}_{v^2} - \left(\frac{v-1}{2v(v-2)}\right)\boldsymbol{E}_{v^2} - \left(\frac{v-3}{2v(v-2)}\right)\boldsymbol{M}_{v^2}. \quad (3.2)$$

Here $\boldsymbol{I}_{v^2}$ is an identity matric of order v^2, $\boldsymbol{E}_{v^2}$ and $\boldsymbol{M}_{v^2}$ are called association matrices and are defined as follows: $\boldsymbol{E}_{v^2} = \{e_{jl}\}$ is a symmetric matrix of order v^2 with elements 0's and 1's where $e_{jl} = 1$ if the j^{th} and l^{th} tree-crop combinations are first associates and $e_{jl} = 0$ otherwise, and $\boldsymbol{M}_{v^2} = \{m_{jl}\}$ is a symmetric matrix of order v^2 with elements 0's and 1's where $m_{jl} = 1$ if the j^{th} and l^{th} tree-crop combinations are second associates and $m_{jl} = 0$ otherwise. It can be seen clearly that, with the exception of the main diagonal elements in $\boldsymbol{E}$ and $\boldsymbol{M}$ where $e_{jj} = m_{jj} = 0$, any pair e_{jl} and $m_{j'l'}$ are mutually exclusive such that when $e_{jl} = 1$, then $m_{j'l'} = 0$ and vice versa for all $j = j'$ and $l = l'$. The association matrices $\boldsymbol{E}$ and $\boldsymbol{M}$ satisfy the following conditions: $\boldsymbol{E}_{v^2}\boldsymbol{1}_{v^2} = (v-1)\boldsymbol{1}_{v^2}$, $\boldsymbol{1}_{v^2}'\boldsymbol{E}_{v^2} = (v-1)\boldsymbol{1}_{v^2}'$, $\boldsymbol{M}_{v^2}\boldsymbol{1}_{v^2} = v(v-1)\boldsymbol{1}_{v^2}$ and $\boldsymbol{1}_{v^2}'\boldsymbol{M}_{v^2} = v(v-1)\boldsymbol{1}_{v^2}'$.

Similarly, $\boldsymbol{C}_{net} = \boldsymbol{C}_{22} - \boldsymbol{C}_{21}\boldsymbol{C}_{11}^{-}\boldsymbol{C}_{12}$, where $\boldsymbol{C}_{net}$ is the information matrix for estimation of tree network effects and $\boldsymbol{C}_{11}^{-}$ is the generalized inverse of $\boldsymbol{C}_{11}$. On simplification

$$\boldsymbol{C}_{net} = \frac{4v^2(v-3)}{(v-1)}\left[\boldsymbol{I}_v - \frac{1}{v}\boldsymbol{J}_v\right]. \quad (3.3)$$

From Section 2.1, the first v^2 columns of $\boldsymbol{X}_1$ correspond to the direct effects of the v^2 tree-crop combinations and the last v columns correspond to the network effects of the v tree species. We recall that $\boldsymbol{Q}$ is a (v^2+v)-column vector of which the first v^2 elements correspond to direct effects and the last v elements correspond to network effects. Therefore, we have $\boldsymbol{Q}' = [\boldsymbol{Q}_1' \quad \boldsymbol{Q}_2']$ where $\boldsymbol{Q}_1'$ is a v^2-vector and $\boldsymbol{Q}_2'$ is a v-vector. Now the direct effects of tree-crop combination may be estimated as $\hat{\boldsymbol{\tau}} = \boldsymbol{C}_{dir}^{-}\boldsymbol{Q}_1$ and the network effects of tree species as $\hat{\boldsymbol{\delta}} = \boldsymbol{C}_{net}^{-}\boldsymbol{Q}_2$, where $\boldsymbol{C}_{dir}^{-}$ and $\boldsymbol{C}_{net}^{-}$ are the generalized inverses of $\boldsymbol{C}_{dir}$ and $\boldsymbol{C}_{net}$ respectively and $\boldsymbol{Q} = \boldsymbol{X}_1'\left(\boldsymbol{I}_{v^2} - \frac{1}{n}\boldsymbol{J}_{v^2}\right)\boldsymbol{Y}$ is the vector of totals of tree-crop combinations.

It can be seen from $\boldsymbol{C}_{dir}$ in Equation (3.2) that the tree-crop combinations are grouped into first and second associates, and different sets of tree-crop combinations are estimated with different variances. Thus the elementary contrasts pertaining to any pair of tree-crop combinations that are first associates is estimated with the same variance. Also, the elementary contrasts among tree-crop combinations that are second associates are estimated with a constant variance. This implies that this series comprises designs that are partially variance balanced with tree-crop combinations following a two-associate class Group Divisible (GD) association scheme. From Example 1 in Section 2.2, it can easily be shown that the design has 25 tree-crop combinations which follow the GD association scheme. For instance, the tree-crop combination 1a has its set of first associates as {1b, 1c, 1d, 1e} and set of second associates as {0a, 0b, 0c, 0d, 0e, 2a, 2b, 2c, 2d, 2e, 3a, 3b, 3c, 3d, 3e, 4a, 4b, 4c, 4d, 4e}. The joint information matrix for this design is obtained as:

$$C_{\theta_1} = \begin{bmatrix} 2\left[I_{25} - \frac{1}{25}J_{25}\right] & -2\left[I_5 - \frac{1}{5}J_5\right] \otimes 1_5 \\ -2\left[I_5 - \frac{1}{5}J_5\right] \otimes 1_5' & 60\left[I_5 - \frac{1}{5}J_5\right] \end{bmatrix}.$$

The corresponding information matrices for estimation of direct effects of tree-crop combination and tree network effects are obtained as:

$C_{dir} = \frac{28}{15} I_{25} - \frac{2}{15} E_{25} - \frac{1}{15} M_{25}$ and

$C_{net} = 10[5I_5 - J_5]$, respectively.

3.2 Canonical efficiency factor

The efficiencies of the proposed designs are assessed using canonical efficiency factors. We require the eigenvalues of C_{dir} to calculate the canonical efficiency factors. There are a total of $(v^2 - 1)$ non-zero eigenvalues which can be put into two sets. Let λ_s denote the s^{th} $(s = 1, 2, \ldots, v^2 - 1)$ eigenvalue, then from C_{dir}, the first set of eigenvalues is obtained as $\lambda_1 = (v-1)/2$ with multiplicity $v(v-1)$ and the second set is $\lambda_2 = v(v-3)/(2(v-2))$ with multiplicity $(v-1)$. We know that the tree-crop combinations are equi-replicated r_1 times. The canonical efficiency (CE) factor of the proposed designs relative to an orthogonal design with the same number of tree-crop and replications has been computed by working out $(1/r_1)$ times the harmonic mean of the eigenvalues (Dey, 2008). Therefore CE is obtained as:

$$CE = \frac{v(v^2-1)(v-3)}{(v-1)(v^3-2v^2-3v+2)}. \tag{3.4}$$

The SAS code for calculation of the information matrices, eigenvalues and CE factors is attached in Appendix II. The CE has been computed for designs in the range $5 \leq v \leq 19$, and given in Table 2. The CEs are very high and show an increasing trend when number of tree species increases. This implies that the proposed designs are highly efficient for estimation of direct effects of tree-crop combinations (Table 2). Efficient designs serve as improved statistical methodology for statistical analyses through reduction of estimated experimental error variance. Such improvement is achievable through appropriate arrangement of the plots as well as plot size as the experimental situation may require. Though small plots are usually preferred in most agricultural field trials, it may not be the case in agroforestry owing to the involvement of multiple components, especially trees, which usually require larger area. For instance, if small-sized plots are used, the canopy of established trees might hinder the growth of crops on such plots.

The proposed designs would be appropriate for multi-tree multi-crop species trials. The separate arrays permit for their implementation in multiple locations. Besides the simplicity in construction, the designs could be advantageously used to produce more precise estimates of tree-crop effects in agroforestry trials. It is worth noting that the designs attain general balance only when $t = c = v$, thereby limiting the total tree-crop combinations to v^2.

Table 2. Canonical efficiency factors of TNetBD

Serial No.	v	CE
1	5	0.9677
2	7	0.9912
3	11	0.9981
4	13	0.9989
5	17	0.9995
6	19	0.9997

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APPENDIX I

SAS code to analysis total tree-crop yield from TNetBD

Note: The data is prepared for imputation into SAS by coding in the following manner, dir is coded as 0a=1, 0b=2, 0c=3, …, 4e=25; net is coded as 0=1, 1=2, …, 4=5.

```
datatcnet;
inputdir tree crop   net    obs;
datalines;
1   1   1   2   62.7984
1   1   1   5   62.7984
1   1   1   3   64.5112
1   1   1   4   64.5112
2   1   2   2   70.4031
2   1   2   5   70.4031
2   1   2   3   68.6204
2   1   2   4   68.6204
3   1   3   2   68.7125
3   1   3   5   68.7125
 …
   …
   …
24  5   4   3   73.475
24  5   4   2   73.475
25  5   5   4   64.316
25  5   5   1   64.316
25  5   5   3   64.669
25  5   5   2   64.669
;
procglmdata=tcnet;
classdir net;
modelobs = dir net /ss1;
*means dir;
*lsmeansdir;
run;

procglmdata=tcnet;
class tree crop net;
modelobs = tree crop(tree) net /ss1;
*means dir;
*lsmeansdir;
run;

procglmdata=tcnet;
classdir net;
modelobs = net dir /ss1;
*means dir;
*lsmeansdir;
run;

procglmdata=tcnet;
class tree crop net;
modelobs = net tree crop(tree) /ss1;
*means dir;
*lsmeansdir;
run;

quit;
```

APPENDIX II

SAS code for information matrices, eigenvalues and canonical efficiency factors

```
/*ENTER THE DESIGN WITHOUT BORDERS (It will take circular border)*/
*odscsv file=”F:\Eldho\Peter\temp13.csv” ;
%letsq= ;
prociml;
/*enter design without borders*/
```

```
a_d={

};/*Trees*/

T_C={

}; /*Tree and Crop*/

dokk=1to&sq;

a=j(nrow(a_d), ncol(a_d)/&sq,0);
do i=1tonrow(a_d);
k=1;
do j=(ncol(a_d)/&sq)*(kk-1)+1to
(ncol(a_d)/&sq)*(kk);
a[i,k]=a_d[i,j];
k=k+1;
end;

end;

*print a;

/***********CIRCULAR Borders*********/
aa=a[ ,ncol(a)]||a||a[ ,1];

*print a01;

aaa=a[nrow(a), ]//a//a[1, ];

*print aa;
/*************************************/

/***********SELF Borders*********/
*aa=a[ ,1]||a||a[ ,ncol(a)];

*print a01;

*aaa=a[1, ]//a//a[nrow(a), ];

*print aa;
/*************************************/

a0=j(nrow(a),ncol(a),0);
do i=1tonrow(a);
do j=1toncol(a);
a0[i,j]=(i-1)*ncol(a)+j;
end;
end;
*print a0;

a01=a0[ ,ncol(a0)]||a0||a0[ ,1];

*print a01;

a1=a0[nrow(a01), ]//a0//a0[1, ];

*print a1;

adj_mat=j(nrow(a)*ncol(a),nrow(a)*ncol(a),0);
/*Adjacency matrix*/
k=1;
do i=1tonrow(a);
do j=2toncol(a)+1;
ifaa[i,j-1]>0thenadj_mat[k,a01[i,j-1]]=1;
k=k+1;
end;
end;

k=1;
do i = 1tonrow(a);
do j = 2toncol(a)+1;
ifaa[i,j+1]>0
    then do;
adj_mat[k,a01[i,j+1]]=1;
end;
k=k+1;
end;
end;

k=1;
do i = 2tonrow(a)+1;
do j = 1toncol(a);
ifaaa[i-1,j]>0
    then do;
adj_mat[k,a1[i-1,j]]=1;
```

```
end;
k=k+1;
end;
end;

k=1;
do i = 2tonrow(a)+1;
do j = 1toncol(a);
ifaaa[i+1,j]>0
    then do;
adj_mat[k,a1[i+1,j]]=1;
end;
k=k+1;
end;
end;

*print adj_mat;

*if kk=1 then do;
*adj_mat1=adj_mat;
*end;

adj_mat1=adj_mat1//adj_mat;

end;

*print adj_mat1;

*adj_mat2 = block(adj_mat1[1:15,],adj_mat1[16:30,],adj_mat1[31:45,], adj_mat1[46:60,]);/* one has to manually change it */

adj_mat2=j(nrow(a_d)*ncol(a_d),nrow(a_d)*ncol(a_d),0);
do i=1to&sq;
do j=(i-1)*ncol(adj_mat1)+1to i*ncol(adj_mat1);
do k=1toncol(adj_mat1);
adj_mat2[j,k+((i-1)*(ncol(adj_mat1)))]=adj_mat1[j,k];
end;
end;
end;
*print adj_mat2;

U=j(nrow(a_d)*ncol(a_d),max(a_d),0);/*design matrix -obs VS direct treatment*/
k=1;
dokk=1to&sq;
do i=1tonrow(a_d);
do j=(ncol(a_d)/&sq)*(kk-1)+1to (ncol(a_d)/&sq)*(kk);
ifa_d[i,j]>0
then U[k,a_d[i,j]]=1;
k=k+1;
end;
end;
end;
*print U;

TC=j(nrow(t_c)*ncol(t_c),max(t_c),0);/*design matrix -obs VS T and C*/
k=1;
dokk=1to&sq;
do i=1tonrow(t_c);
do j=(ncol(t_c)/&sq)*(kk-1)+1to (ncol(t_c)/&sq)*(kk);
ift_c[i,j]>0
then TC[k,t_c[i,j]]=1;
k=k+1;
end;
end;
end;
*print TC;

UU=j(nrow(U),1,1);
AU=adj_mat2*U;
F=TC||AU||UU;

*print AU;
Z=F`*F;

X1=TC||AU;
```

```
X2=UU;

C_Matrix=(x1`*x1)-(x1`*x2*(ginv(x2`*x2))*x2`*x1);

*print adj_mat2;
*print Z;
*print AU;
printC_Matrix;

c11=j(max(T_C),max(T_C),0);
do i=1to max(T_C);
do j=1to max(T_C);
c11[i,j]=C_Matrix[i,j];
end;
end;
*print c11;

c12=j(max(T_C),max(a_d),0);
do i=1to max(T_C);
k=1;
do j=max(T_C)+1toncol(C_Matrix);
c12[i,k]=C_Matrix[i,j];
k=k+1;
end;
end;
*print c12;

c22=j(nrow(C_Matrix)-max(T_C),nrow(C_Matrix)-
max(T_C),0);
k=1;
do i=max(T_C)+1tonrow(C_Matrix);
kk=1;
do j=max(T_C)+1tonrow(C_Matrix);
c22[k,kk]=C_Matrix[i,j];
kk=kk+1;
end;
k=k+1;
end;
*print c22;
c_direct_tree=c11- c12*ginv(c22)*c12`;
c_network=c22- c12`*ginv(c11)*c12;

rank_c11 = round (trace(ginv(c11)*c11));
rank_c12 = round (trace(ginv(c12)*c12));
rank_c22 = round (trace(ginv(c22)*c22));

printc_direct_tree;
printc_network;

*print c11;
*print c12;
*print c22;

eigD=eigval(c_direct_tree);
eigN=eigval(c_network);
printeigDeigN;
eigD1=eigD[loc(eigD>0.0000001),];/*positive
eigen values*/
eigN1=eigN[loc(eigN>0.0000001),];/*positive
eigen values*/
eigD2=eigD1/Z[1,1];
eigD3=1/eigD2;
CanEffFacD=nrow(eigD3)/sum(eigD3);
printCanEffFacD;

run;

*ods rtf close;
quit;
```