International Journal of Ecology and Environmental Sciences
www.ecologyjournal.in
Online ISSN: 2664-7133; Print ISSN: 2664-7125; Impact Factor: RJIF 5.28
Received: 08-12-2020; Accepted: 24-12-2020; Published: 06-01-2021
Volume 3; Issue 1; 2021; Page No. 36-41

Robust designs involving partial triallel crosses for breeding experiments

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Abstract
Obtaining information regarding general combining ability (Gca) and specific combining ability (sca) effects can be the major objective of a breeding programme to release new hybrids with enhanced genetic potential. This study included designs involving partial triallel crosses as they, being genetically more viable and consistent performers, involve lesser number of crosses leading to a lower degree of fractionation. An optimal or efficient design for triallel cross experiment may become disconnected and inefficient if the underlying assumptions are not fulfilled due to a missing observation pertaining to a cross. The robustness of designs against missing observation using connectedness and efficiency criteria has been studied both under unblocked and blocked situations. A list of efficient robust designs for triallel cross experiments has been tabulated for both unblocked and blocked situation. Programs have been written in SAS [PROC IML] software for computing efficiency factor of the designs involving triallel crosses for estimating Gca effects to investigate the robustness of designs against missing observation by calculating the efficiency factor.

Keywords: connectedness; degree of fractionation; efficiency factor; general combining ability; partial triallel cross; specific combining abilities

Introduction

Breeding techniques are used as a tool for the development of commercial hybrids for which a major objective of plant and animal breeders is to raise the genetic potential. Any breeding experiment centres on acquiring information regarding the combining ability effects. The information collected on gca and sca forms a basis of making correct choice of the best parental lines. One of the most common and rigorously used breeding techniques is diallel crossing as they are simple and easily manageable. However, higher order crosses like triallel cross based hybrids are genetically more viable, stable and consistent in performance than diallel cross hybrids. Triallel cross hybrids have wider genetic base which gives them strong buffering mechanism as individual or when constituting a population.

There are many cases of crops (like maize or corn) and animals (like swine and chicken) where triallel crosses are used for producing commercial hybrids (Shunmuguthai and Srinivasan 2012) [25]. Triallel crossbred chickens show better egg traits than diallel crossbred chickens and are also having lower mortality (Khawaja *et al.* 2013) [15]. Triallel crossing scheme is very much acceptable and practiced in pig farming. The resultant product is also economical and of good quality. The silkworm production industry is also practicing the triallel crosses for exploitation of heterosis (Das *et al.* 1997) [5]. Applications of triallel cross experiment has been also done in groundnut (Varman and Thangavelu 1999) [26]. Triallel crosses, often referred as three-way crosses, are those type of mating designs in which each cross is obtained by crossing three inbred lines. A triallel cross can be obtained by crossing the resultant of a diallel cross with an unrelated inbred line. A common triallel cross involving three inbred lines A, B and C can be symbolically represented as (A×B)×C or (A, B, C) or simply (A B C). Unlike diallel cross, the three lines involved in the triallel cross do not contribute equally and thus, it is important to differentiate amongst them. The two lines A and B which are used first to produce a diallel cross contribute half as much as that of the third line C used to obtain the triallel cross. Hence, lines A and B are also referred as half parents whereas line C as full parent. Triallel crosses can be broadly categorized as complete triallel cross (CTC) and partial triallel crosses (PTC). The set of all possible three-way matings between several genotypes (individuals, clones, homozygous lines, *etc*) leads to a CTC. Triallel cross has been defined by Rawlings and Cockerham (1962) [23] as a set of all possible distinct three-way matings among a group of lines. The definition given by them is applicable for CTC. If there are N number of inbred lines involved in a CTC, the total number of crosses is

$$T = \frac{N(N-1)(N-2)}{2}.$$

When the number of lines increases, the total number of crosses in CTC also increases. It is almost impossible for the investigator to handle it with limited available resources. This situation lies in taking a fraction of CTC with certain underlying properties, known as PTC. Hinkelmann (1965) [14] defined PTC as a set of triallel matings in which every line occurs r_h and r_f times as half-parent and full-parent, respectively and each cross of the type (A×B)×C {alongwith (B×C)×A and (A×C)×B, to maintain the Structural Symmetry Property (SSP)} occurs either once or not at all. The total number of crosses is N times r_f. Here is an example of PTC consisting of 63 crosses that can be made for 7 lines (A, B, C, D, E, F and G) with r_f = 9, r_h = 18, f = 3/5 (Note that, the degree of fractionation f is defined as the ratio of crosses in a PTC to a CTC for a given number of lines):

Table 1

(A×B)×C	(A×D)×B	(B×C)×A	(A×B)×E	(A×E)×B	(B×E)×A
(A×B)×G	(A×G)× B	(B×G)×A	(A×C)×E	(A×E)×C	(C×E)×A
(A×C)×F	(A×F)×C	(C×F)×A	(A×D)×E	(A×E)×D	(D×E)×A
(A×D)×F	(A×F)×D	(D×F)×A	(A×D)×G	(A×G)×D	(D×G)×A
(A×F)×G	(A×G)×F	(F×G)×A	(B×C)×D	(B×D)×C	(C×D)×B
(B×C)×F	(B×F)×C	(C×F)×B	(B×D)×F	(B×F)×D	(D×F)×B
(B×D)×G	(B×G)×D	(D×G)×B	(B×E)×F	(B×F)×E	(E×F)×B
(B×E)×G	(B×G)×E	(E×G)×B	(C×D)×E	(C×E)×D	(D×E)×C
(C×D)×G	(C×G)×D	(D×G)×C	(C×E)×G	(C×G)×E	(E×G)×C
(C×F)×G	(C×G)×F	(F×G)×C	(D×E)×F	(D×F)×E	(E×F)×D
(E×F)×G	(E×G)×F	(F×G)×E			

Ponnuswamy (1971) [20] considered the research problem of constructing incomplete block designs for triallel crosses. Arora and Aggarwal (1984) [1] discussesd the applications of confounded triallel experiments which is nothing but directly related to the PTC. Arora and Aggarwal (1989) [2] extended their research work on triallel crosses including reciprocal effects in the model. Ceranka *et al.* (1990) [4] have worked regarding the estimation of parameters involved in the model for triallel crosses under the blocked set up. Ponnuswamy and Srinivasan (1991) [21] used a new class of balanced incomplete block (BIB) designs known as Partially Doubly Balanced Incomplete Block (PDBIB) designs for the construction of a class of PTC. Das and Gupta (1997) [5] worked in the area of optimality of block designs for triallel crosses. Sharma *et al.* (2012) [24] considered the problem of investigating optimal class of designs involving PTC. Harun (2014) [9] and Harun *et al.* (2016a, 2016b, 2016c) [10, 11, 12] discussed various methods of constructing designs for partial triallel cross experiments. An optimal or efficient design may not remain so and may become disconnected and all the contrasts pertaining to combining ability effects may not be estimable or may become inefficient if the underlying assumptions are not fulfilled due to disturbances like missing observation(s), outlying observation(s), exchange or interchange of crosses, inadequacy of assumed model, *etc*. The concept of connectedness criterion of robustness was introduced by Ghosh (1978) [8], with respect to robustness of BIB designs. Panda (2000) [18] considered the problem of interchange of a pair of crosses while conducting a complete diallel cross (CDC) experiment. Panda *et al*. (2001) [19] have considered optimal block designs for triallel cross experiments for investigating robustness against an exchanged cross. Dey *et al.* (2001) [7] considered the problem of missing observations in diallel cross experiments. Bhar and Gupta (2002) [3] considered the problem of missing observations in diallel cross experiments. Lal and Jeisobers (2002) [16] investigated the problem of missing crosses from a block to study the robustness of diallel cross designs under blocked set up. Prescott and Mansson (2004) [22] investigated the problem of loss of one or more observations in a diallel cross design. Shunmugathai and Srinivasan (2012) [25] studied robustness of NBIB designs under the situation of interchange of a pair of crosses in triallel crosses. In case of breeding experiments the loss of observation is much prevalent because an observation may not sprout or may not survive till the time of measurement. Besides this, any human error regarding tagging may also result in loss of observation. In this study robust designs involving triallel crosses against missing observation have been obtained using the robustness criteria of connectedness and efficiency. These connected and efficient designs will be helpful for the breeders to estimate the gca effects of lines even if an observation is missing.

Material and methods

Model and experimental setup

Full model

Consider a triallel cross experiment involving N number of lines giving rise to T number of crosses. Let a cross of type $(i \times j) \times k$ is represented as (i, j, k) and the fixed effect of the triallel cross (i, j, k) by y_{ijk}, then the following model can be used for representing cross effect:

$$y_{ijk} = \bar{y} + h_i + h_j + g_k + s_{ij} + s_{ik} + s_{jk} + s_{ijk} + e_{ijk}, \quad (1)$$

where $\bar{y}$ is the average effect of the crosses, $\{h_\alpha\}, \alpha = i, j$ and $\{g_k\}$ represents the gca effects half parents and full parents respectively, $\{s_{\alpha\beta}\}, (\alpha, \beta) \in (i, j, k)$ represents the first order sca effects, s_{ijk} represents the second order sca effects, e_{ijk} represents the random error component with the constraints

$$\sum_{i=1}^{N} h_i = 0 \text{ and } \sum_{i=1}^{N} g_i = 0$$

$$\sum_{\alpha\beta} s_{\alpha\beta} = 0 \; \forall \; (\alpha, \beta) \in (i, j, k), i \neq j \neq k = 1,2, \dots, N \text{ and}$$

$$\sum_{ijk} s_{ijk} = 0 \; \forall \; i \neq j \neq k = 1,2, \dots, N$$

It is important to note here that if a cross (i, j, k) is occurring in the experiment then the other two alternative forms (i, k, j) and (j, k, i) are also included in the experiment, to satisfy the SSP of triallel crosses.

Reduced model

In this approach, gca effects of first and second kind corresponding to half and full parents will be estimated for which it is assumed that the sca effects are contributing much less to the total combining ability effects as compared to gca effects and hence sca effects are negligible. The model can be written as

$$y_{ijk} = \bar{y} + h_i + h_j + g_k + e_{ijk}, \quad (2)$$

where $\bar{y}$ is the average effect of the treatments, $\{h_\alpha\}, \alpha = i, j$, represents the gca effects of first kind corresponding to the lines occurring as half parents, $\{g_k\}$ represents the gca effects of second kind corresponding to the lines occurring as full parents, e_{ijk} is the random error component and

$$g_1 + g_2 + \dots + g_N = 0 \text{ or } \sum_{i=1}^{N} g_i = 0$$

$$h_1 + h_2 + \dots + h_N = 0 \text{ or } \sum_{i=1}^{N} h_i = 0$$

The model in matrix notation is expressed as:

$$\boldsymbol{y} = \bar{y}\, \boldsymbol{1}_T + \boldsymbol{W}_1' \boldsymbol{h} + \boldsymbol{W}_2' \boldsymbol{g} + \boldsymbol{e}, \quad (3)$$

where, $\boldsymbol{y}$ is the $T \times 1$ vector of responses due to crosses, $\bar{y}$ is the mean effect of crosses, $\boldsymbol{h}$ is the $N \times 1$ vector of gca effects due to half parent, $\boldsymbol{g}$ is the $N \times 1$ vector of gca effects due to full parent and $\boldsymbol{e}$ is the $N \times 1$ vector of random error component. $\boldsymbol{W_1}$ and W_2 are $N \times T$ matrices with rows indexed by the line numbers $1,2,\ldots N$ and columns by the three-way crosses arranged in the manner described earlier, such that the $\{t,(i,j,k)\}^{th}$entry of W_1 is 0.5 if $t \in (ij)$ and zero otherwise and the $\{t,(i,j,k)\}^{th}$entry of W_2 is 1 if $t \in k$ and zero otherwise. The normal equations are as

$$E(\boldsymbol{y}) = \bar{y}\,\boldsymbol{1}_T + \boldsymbol{W}_1'\boldsymbol{h} + \boldsymbol{W}_2'\boldsymbol{g},$$

$$\boldsymbol{W_1}E(\boldsymbol{y}) = \bar{y}\,\boldsymbol{W_1}\boldsymbol{1}_T + \boldsymbol{W_1}\boldsymbol{W}_1'\boldsymbol{h} + \boldsymbol{W_1}\boldsymbol{W}_2'\boldsymbol{g},$$ and

$$\boldsymbol{W_2}E(\boldsymbol{y}) = \bar{y}\,\boldsymbol{W_2}\boldsymbol{1}_T + \boldsymbol{W_2}\boldsymbol{W}_1'\boldsymbol{h} + \boldsymbol{W_2}\boldsymbol{W}_2'\boldsymbol{g}.$$

On solving the three normal equations, the estimate of the Gca effects of half parent is given as:

$$\hat{\boldsymbol{h}} = (\boldsymbol{W_1}\boldsymbol{W}_1')^{-}(\boldsymbol{W_1}\boldsymbol{y} - \boldsymbol{W_1}\bar{y}\,\boldsymbol{1}_T)$$

$$= [(\boldsymbol{W_1}\boldsymbol{W}_1')^{-}\boldsymbol{W_1} - (\boldsymbol{W_1}\boldsymbol{W}_1')^{-}\boldsymbol{W_1}\boldsymbol{J}_{TT}/T)]\boldsymbol{y} = \boldsymbol{G}_1\boldsymbol{y} \text{ (say)},$$ (Say), and the estimate of Gca effects of full parent is given as:

$$\hat{\boldsymbol{g}} = (\boldsymbol{W_2}\boldsymbol{W}_2')^{-}(\boldsymbol{W_2}\boldsymbol{y} - \boldsymbol{W_2}\bar{y}\,\boldsymbol{1}_T)$$

$$= [(\boldsymbol{W_2}\boldsymbol{W}_2')^{-}\boldsymbol{W_2} - (\boldsymbol{W_2}\boldsymbol{W}_2')^{-}\boldsymbol{W_2}\boldsymbol{J}_{TT}/N)]\boldsymbol{y} = \boldsymbol{G}_2\boldsymbol{y}$$ (say).

The restrictions being imposed in order to estimate the Gca effects of half parents free from gca effects of full parents are as:

$$\boldsymbol{1}'\hat{\boldsymbol{h}} = \boldsymbol{1}'\hat{\boldsymbol{g}} = \boldsymbol{G}_1\boldsymbol{1} = \boldsymbol{G}_2\boldsymbol{1} = \boldsymbol{G}_1'\boldsymbol{G}_2 = \boldsymbol{0}$$

and

$$rank(\boldsymbol{G_1}) = rank(\boldsymbol{G_2}) = (N-1).$$

Now, considering the usual setup of a block Design d, the joint information matrix regarding $\begin{pmatrix} G_1 \\ G_2 \end{pmatrix} y$ is given by:

$$\boldsymbol{C}_{d\_gca} = \begin{bmatrix} \boldsymbol{G}_1\boldsymbol{C}_d\boldsymbol{G}_1' & \boldsymbol{G}_1\boldsymbol{C}_d\boldsymbol{G}_2' \\ \boldsymbol{G}_2\boldsymbol{C}_d\boldsymbol{G}_1' & \boldsymbol{G}_2\boldsymbol{C}_d\boldsymbol{G}_2' \end{bmatrix},$$

Where $C_d = R_d - \frac{1}{k}N_d N_d'$, $R_d = diag(r_1, r_2, \ldots, r_T)$ is the diagonal matrix of replications of the crosses under the design d and N_d is the incidence matrix of crosses versus blocks. Here, $\boldsymbol{C}_d$ is the information matrix of the general block design d where treatments are nothing but the T number of tri-allele crosses, hence we have $C_d 1_T = 0$. As discussed earlier regarding orthogonality, in order to estimate $G_1 C_d G_1'$ and $G_2 C_d G_2'$ orthogonally the off diagonal components must vanish and we must have $G_2 C_d G_1' = G_1 C_d G_2' = 0$. Thus we have $C_{\text{gca\_half}} = G_1 C_d G_1'$ and $C_{\text{gca\_full}} = G_2 C_d G_2'$.

Designs for PTC experiments

PTC plans with crosses arranged in blocks

Method 1

An easy and general method to obtain a series of partial three-way cross plan using Mutually Orthogonal Latin Squares (MOLS) has been explained by Harun *et al.* (2016a) [10]. Let *N*, the number of lines be a prime number. Construct a complete set of MOLS for *N* using symbols 1, 2…, *N*. Retain first 3 rows of each array of size *N* × *N* corresponding to each of the *N*-1 Latin squares. Thus, *N* - 1 blocks each consisting of 3*N* crosses can be obtained easily by making all possible distinct crosses within each of the array. The parameters of this class of designs are: total number of crosses (*T*) = 3*N* (*N*-1), number of blocks (*b*) = (*N*-1), block size (*k*) = 3*N* and degree of fractionation (*f*) = 6/ (*N*-2).

Method 2

Another series of partial three-way cross plans with crosses arranged in incomplete blocks has been obtained by Harun *et al.* (2016a) [10] using MOLS. Let *N*, the number of lines be a prime number. Construct a complete set of MOLS for *N* using symbols 1, 2, …, *N* and choose any of the (*N*-1)/2 Latin squares from the complete set of MOLS. Retain first 3 rows of each array of size *N* × *N* corresponding to each of the (*N*-1)/2 Latin squares. Thus, (*N*-1)/2 blocks each consisting of 3*N* crosses can be obtained easily by making all possible distinct crosses within each of the array. The parameters of this class of designs are: total number of crosses (*T*) = 3*N* (*N*-1)/2, number of blocks (*b*) = (*N*-1)/2, block size (*k*) = 3*N* and degree of fractionation (*f*) = 3/ (*N*-2).

PTC plans using partially balance incomplete block (PBIB) designs

Harun *et al.* (2016a) [10] considered PBIB designs with parameters, $v^*, b^*, r^*, k^*, \lambda^*$, having small block size ($k^* > 2$) for constructing PTC plans. Considering the symbols/ numerals of the block contents as lines and making all possible three-way crosses within each block, we can obtain a partial three-way cross plan. Total number of crosses in this class of designs are $T = b^*k^*(k^*-1)(k^*-2)/6$.

PTC plans using Kronecker product

This method was given by Harun (2014) [9] to obtain PTC plans for composite number of lines. In this method the incidence matrices (N_1 and N_2) of any two BIB designs with number of lines v_1 and v_2 respectively are considered. The Kronecker product ($N_1 \otimes N_2$) of these two matrices is obtained which is the incidence matrix of another PBIB design. Now, from each block of the resultant PBIB design all possible triplet combinations are taken to make three-way crosses constituting a PTC plan with $N = v_1 v_2$.

PTC plans using triangular association scheme

Another class of PTC plans with crosses arranged in blocks has been obtained by Harun *et al.* (2019) [13]. Let there be $N = \frac{n(n-1)}{2}$ lines, Where $n > 4$. Arrange these N lines in a two-associate triangular association scheme. Diagonal positions are left empty. Consider all possible pair of lines.

that can be made from each row of the array. Add a third line to each of these pairs to form triplets. Line that appears at the intersection of the second row containing the first line in the pair and column containing the second line in the pair is considered, and added to each pair to form triplets. Make three-way crosses from these triplets considering lines in the pairs as half parents and third added line in the triplet as full parent. This will result in a PTC design with parameters $N = \frac{n(n-1)}{2}$, $T = \frac{n(n-1)(n-2)}{2}$, $b = n$, $k = \frac{(n-1)(n-2)}{2}$, $r_h = 2(n-2)$ and $r_f = (n-2)$.

Robust designs for PTC experiments

An optimal or efficient design for triallel cross experiment may not allow the estimation of all elementary contrasts pertaining to gca effects of lines or may become inefficient due to missing observation corresponding to a cross. Hence, it is much important to obtain robust designs against missing observation. The connectedness and efficiency criteria of robustness have been considered here to characterize robust designs involving triallel crosses. Thus, a design for breeding experiments is said to be robust (considering the connectedness and efficiency criteria) against a missing observation, if remains connected and efficient even after the disturbances due to a missing observation.

Let the original design be denoted as d and the residual design after the missing observation as d^*. Let $\mathbf{C}_{d_{gca-half}}$ and $\mathbf{C}_{d_{gca-full}}$ are the information matrices related to half parents and that of full parents respectively under the original design d and $\mathbf{C}_{d^*{}_{gca-half}}$ and $\mathbf{C}_{d^*{}_{gca-full}}$ are the information matrices related to half parents and that of full parents for the residual design d^*.

Criterion of connectedness

A design d involving triallel crosses is said to be robust against missing observation based on connectedness criterion if the residual design d^* remains connected so that we can estimate all the elementary contrasts pertaining to the

- gca effect of first kind *i.e.* gca effect of half parents, and
- gca effect of second kind *i.e.* gca effect of full parents.

Thus a design d^* will be fulfilling this criterion if $\boldsymbol{h}^*$ and $\boldsymbol{g}^*$ representing the new set of orthogonal contrasts are having $(N - 1)$ degrees of freedom.

Efficiency criterion

A robust design involving triallel cross must be efficient pertaining to the gca effects of half as well as full parents.

Efficiency criterion for half parents

The efficiency of the design d^* in comparison to the original design d can be calculated as

$$E_h = \frac{\text{harmonic mean of non-zero eigenvalues of } \mathbf{C}_{d^*{}_{gca-half}}}{\text{harmonic mean of non-zero eigenvalues of } \mathbf{C}_{d_{gca-half}}}.$$

Efficiency criterion for full parents

The efficiency of the design d^* in comparison to the original design d can be calculated as

$$E_f = \frac{\text{harmonic mean of non-zero eigenvalues of } \mathbf{C}_{d^*{}_{gca-full}}}{\text{harmonic mean of non-zero eigenvalues of } \mathbf{C}_{d_{gca-full}}}.$$

List of robust PTC designs

The designs for PTC plans have been investigated for robustness using the connectedness and efficiency criteria and a list of robust designs against missing observation which are connected and are having efficiency more than equal to 80 % is given in Tables 1 to 4. These tables contain the parameters of the robust designs along with the degree of fractionation, efficiencies and the underlying method of construction used. Two situations of blocked and unblocked setup are considered here. In the first situation, corresponding to the missing cross other two crosses are also omitted to satisfy the structural symmetry property. In the second situation, all the crosses except the missing one are kept intact and the study is carried out.

Results and discussion

Robustness of designs for triallel cross experiments have been investigated using connectedness and efficiency criteria against missing observation under unblocked situation and maintaining the SSP. The designs are connected if the number of lines is more than 5. Moreover, the designs are having good efficiencies for estimating the contrasts pertaining to gca effects of half as well as full parents. A list of robust PTC designs under unblocked situation and maintaining SSP is given here in Table 1. The designs are having low degree of fractionation and high efficiencies.

Table 2: Robust designs against a missing observation with SSP under unblocked situation

S.No.	*N*	*T*	*f*	E_h	E_f	Design/Association scheme/ Method used for construction
1	5	30	1.00	0.84	0.86	MOLS
2	5	60	2.00	0.94	0.94	MOLS
3	7	63	0.60	0.94	0.94	MOLS
4	7	126	1.20	0.98	0.98	MOLS
5	10	30	0.08	0.99	0.92	Triangular Association Scheme
6	10	90	0.25	0.95	0.96	Triangular Design
7	11	165	0.33	0.98	0.98	MOLS
8	11	330	0.67	0.99	0.99	MOLS
9	13	39	0.05	0.99	0.95	Cyclic Design
10	13	234	0.27	0.99	0.99	MOLS
11	13	668	0.78	0.99	0.99	MOLS
12	15	60	0.04	0.85	0.88	Triangular Association Scheme
13	17	408	0.20	0.99	0.99	MOLS
14	17	816	0.40	0.99	0.99	MOLS
15	19	513	0.18	0.99	0.99	MOLS
16	19	1026	0.35	0.99	0.99	MOLS
17	21	105	0.03	0.94	0.95	Triangular Association Scheme
18	23	759	0.14	0.99	0.99	MOLS
19	23	1518	0.29	0.99	0.99	MOLS
20	28	168	0.02	0.97	0.97	Triangular Association Scheme

In another approach, the SSP is neglected and robustness of designs for triallel cross experiments have been investigated using connectedness and efficiency criteria against missing observation under unblocked situation.

The main advantage over the previous one is that the designs can be used for lower number of lines also as they remain connected. Further, the efficiencies are much higher. A list of robust PTC designs under unblocked condition and without considering the SSP is given here in the Table 2. The efficiencies are slightly higher than the previous case where SSP was maintained.

Table 3: Robust designs against a missing observation without SSP under unblocked situation

S.No.	*N*	*T*	*f*	E_h	E_f	Design/Association scheme/ Method used for construction
1	5	15	0.50	0.83	0.94	Cyclic Design
2	5	30	1.00	0.95	0.95	MOLS
3	5	60	2.00	0.98	0.98	MOLS
4	7	63	0.60	0.98	0.98	MOLS
5	7	126	1.20	0.99	0.99	MOLS
6	10	30	0.08	0.92	0.92	Triangular Association Scheme
7	10	90	0.25	0.99	0.98	Triangular Design
8	11	165	0.33	0.99	0.99	MOLS
9	11	330	0.67	0.99	0.99	MOLS
10	13	39	0.05	0.93	0.93	Cyclic Design
11	13	234	0.27	0.99	0.99	MOLS
12	13	668	0.78	0.99	0.99	MOLS
13	15	60	0.04	0.97	0.97	Triangular Association Scheme
14	17	408	0.20	0.99	0.99	MOLS
15	17	816	0.40	0.99	0.99	MOLS
16	19	513	0.18	0.99	0.99	MOLS
17	19	1026	0.35	0.99	0.99	MOLS
18	21	105	0.03	0.98	0.98	Triangular Association Scheme
19	23	759	0.14	0.99	0.99	MOLS
20	23	1518	0.29	0.99	0.99	MOLS
21	28	168	0.02	0.99	0.99	Triangular Association Scheme

Under blocked set up too, robustness of designs for triallel cross experiments with SSP has been investigated using connectedness and efficiency criteria against missing observation. These designs, with smaller block sizes are more appropriate for the situation. A list of robust PTC designs under blocked situation and maintaining SSP is provided here in Table 3. The designs are having low degree of fractionation and higher efficiencies.

Table 4: Robust designs against a missing observation with SSP under blocked situation

S.No.	*N*	*b*	*k*	*T*	*f*	E_h	E_f	Design/Association scheme/ Method used for construction
1	5	2	15	30	1.00	0.81	0.84	MOLS
2	5	4	15	60	2.00	0.93	0.93	MOLS
3	7	3	21	63	0.60	0.93	0.94	MOLS
4	7	6	21	126	1.20	0.97	0.97	MOLS
5	9	9	12	108	0.43	0.95	0.96	Kronecker Product
6	11	5	33	165	0.33	0.98	0.98	MOLS
7	11	10	33	330	0.67	0.99	0.99	MOLS
8	12	18	12	216	0.33	0.98	0.98	Kronecker Product
9	13	6	39	234	0.27	0.98	0.99	MOLS
10	13	12	39	668	0.78	0.99	0.99	MOLS
11	15	30	12	360	0.26	0.99	0.99	Kronecker Product
12	17	8	51	408	0.20	0.99	0.99	MOLS
13	17	16	51	816	0.40	0.99	0.99	MOLS
14	19	9	57	513	0.18	0.99	0.99	MOLS
15	19	18	57	1026	0.35	0.99	0.99	MOLS
16	23	11	69	759	0.14	0.99	0.99	MOLS
17	23	22	69	1518	0.29	0.99	0.99	MOLS

In another approach, the SSP is neglected and robustness of designs for triallel cross experiments have been investigated using connectedness and efficiency criteria against missing observation under blocked situation. A list of robust PTC designs under a blocked situation without maintaining SSP is given here in Table 4. It can be seen that the efficiencies are slightly higher than the case where SSP is maintained.

Table 5: Robust designs against a missing observation without SSP under blocked situation

S.No.	N	b	k	T	f	E_h	E_f	Design/Association scheme/ Method used for construction
1	5	2	15	30	1.00	0.95	0.95	MOLS
2	5	4	15	60	2.00	0.98	0.98	MOLS
3	7	3	21	63	0.60	0.98	0.98	MOLS
4	7	6	21	126	1.20	0.99	0.99	MOLS
5	9	9	12	108	0.43	0.99	0.99	Kronecker Product
6	10	5	6	30	0.08	0.97	0.78	Triangular Design
7	11	5	33	165	0.33	0.99	0.99	MOLS
8	11	10	33	330	0.67	0.99	0.99	MOLS
9	12	18	12	216	0.33	0.99	0.99	Kronecker Product
10	13	6	39	234	0.27	0.99	0.99	MOLS
11	13	12	39	668	0.78	0.99	0.99	MOLS
12	15	30	12	360	0.26	0.99	0.99	Kronecker Product
13	17	8	51	408	0.20	0.99	0.99	MOLS
14	17	16	51	816	0.40	0.99	0.99	MOLS
15	19	9	57	513	0.18	0.99	0.99	MOLS
16	19	18	57	1026	0.35	0.99	0.99	MOLS
17	23	11	69	759	0.14	0.99	0.99	MOLS
18	23	22	69	1518	0.29	0.99	0.99	MOLS

Conclusions

Designs involving triallel cross experiments are needed by the breeders to develop stable hybrids. Robustness of designs for triallel cross experiments have been investigated using connectedness and efficiency criteria against a missing observation under unblocked and blocked setup. These robust designs have been tabulated with efficiency factors. The list of efficient designs with lower degree of fractionation for triallel cross experiments under blocked and unblocked setup can be used as robust designs against a missing observation. Programs have been written in SAS [PROC IML] software which can be used to investigate the robustness of designs against missing observation by calculating the efficiency factor.

Acknowledgement

The first author would like to thank Post Graduate School IARI, New Delhi and Director ICAR-IASRI, New Delhi for providing necessary facilities.

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