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MATHEMATICAL MODELS IN FISH STOCK ASSESSMENT

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ABSTRACT

Fishery resources, being renewable, have to be exploited in such a way that reaping of maximum sustainable yield (MSY) is possible without affecting the stocks. For such an effective exploitation, size of stocks is to be determined for obtaining their MSY. But these resources unlike the other living resources, being not in the visual domain for direct evaluation, methods of their assessment need not be the same as used for others.

In this paper an attempt has been made to enumerate different models used for fish stock assessment. Most of the models till date are deterministic. For unit stock assessment these are considered under Macro and Micro-analytic models. Extension of such models for multispecies is also indicated. Stochastic models, though at infant stage are also available and the same too are presented in this paper.

INTRODUCTION

AMONG the living resources fish stocks occupy an important place in providing cheap protein food, employment and income to millions of people all over the world. No wonder then that these resources attract attention from different spheres of human activity including religion. It may not be surprising to note that wars between countries have been declared on the exploitation of these valuable resources (Royce, 1972). Hence, judicious management and exploitation of these renewable resources is the paramount need of the hour to draw sustainable yields for years to come. In India fishing is an age old occupation. From a modest 0.5 million tonnes per annum in the fifties Indian marine fish landings have gone up to 1.6 million tonnes in the eighties. In the exports too, the foreign exchange earned by India now is over Rs. 400 crores.

Assessment of fish stocks is the first step to determine the required level of exploitation for reaping maximum sustainable yields (MSY). In case of exploited stocks, the impact of the present level of exploitation is also to be gauged for arriving at better management policies for future planning and developmental purposes in order to obtain MSY. Overfishing leads to disappearance of stocks and creation of aqua-deserts so far as those over fished stocks are concerned. Whales of Antarctica stand as an example reminding the mankind the serious lesson learnt from the story of killing the goose laying golden eggs. At the same time one should not forget that the carrying capacity of any water body can be improved by properly fixing the level of exploitation. Apart from this, exploitation of common stocks in waters shared by many countries poses serious problems not only on the stocks, but also on the relationship of neighbouring countries sometimes leading to wars as mentioned earlier. Hence assessment of stocks and the impact of present level of effort exerted on the exploited stocks have to be determined for solving major problems namely maintenance of stocks at levels required for obtaining MSY and upholding the legitimate interests of the countries involved in exploiting common resources. In order to achieve this objective many mathematical models have been developed and used on fish stock assessment studies.

Fish stocks differ from other living resources mainly on three accounts. First of all these stocks do not come under visual horizon for direct evaluation of their sizes, as in the case of any other living resources. Their distribution over space, time and species varies in such a way that resultant variability assumes higher dimensions not encountered in any other resource estimation. Finally, fish stocks, particularly the exploited ones, are affected by fishery dependent and fishery independent factors. Fishery dependent factors include the level of effort exerted on the stocks and the size at first capture. Fishery independent factors, on the other hand, include salinity, temperature, water current, etc. that affect the stocks. Though fishery dependent factors are controllable, fishery independent factors are not controllable and also their resultant effect on the stocks is not easily measurable. This last factor makes forecast studies in fisheries a very difficult one. Viewing from these angles, mentioned above, one would wish that the models to study fish stocks should take into account atleast, the three aspects namely, the size of stocks, present level exploitation and the effect of fishery independent factors. The dimension of the parametric space expected in such models incorporating all the above aspects becomes very large and the wide variability encountered over space and time makes it a formidable task in collection of representative samples to yield precise and valid estimates of the parameters of the mondels. For unit stock assessments assuming equilibrium conditions by which the variabiility due to fishery independent factors are safely ignored, models have been developed and widely used in assessing fish stocks. One more dimension to the problem of assessing fish stocks is added by the presence of multispecies operated upon by multigears so that species specific or gear specific approach in most of the existing models become less useful in assessing fish stocks. Nevertheless, mathematical models suited to fish stock assessments satisfying atleast some of the requirements indicated above have been developed and successfully used to offer suitable management programmes so as to obtain MSY without affecting the stocks. This paper makes an attempt to present these models under different categories. The major category consists of deterministic models. The models are further divided into the groups namely Macro-analytical and Micro-analytical models. Stochastic models incorporating random element in the deterministic models are also presented.

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DETERMINISTIC MODELS

Deterministic models bereft of random element for allowing chance fluctuations in the levels of different parameters are no doubt crude in their approach leading to wider approximation of the existing status of the stocks. As indicated earlier increasing the dimensions in the parametric space of models normally result in more complication and the data base may not be strong enough to support this approach. Hence macro-analytic models are considered for rough approximations of the existing status of any stock. These models normally do not involve more than catch and effort data. Thus macro-analytic models are more simple and collection of data becomes simpler and analysis of data is straight forward. Some of the models, under this group, commonly used in fish stock assessment, are given below.

Macro-analytic Models

Let B_0 and B_1 be the initial and final biomasses of a stock in a year. The change in the biomass is given (Russell, 1931) by

$$B_1 - B_0 = R + G - D \qquad \ldots(1)$$

Where R, G and D denote recruitment, growth and mortality - both natural and fishing respectively. Once D takes care of reduction in the stock due to all factors such as predation and emigration and the factors R and G that of increase due to immigration then (1) completely determines the condition of any stock. Thus (1) may be considered as a general model describing the changes in biomass of any fish stock. In Macro-analytical models otherwise called synthetic models or global models the overall net effect of all factors that control the biomass is considered simultaneously. In this, the relationship of $\triangle B$ ($= B_1 - B_0$) with B the biomass, F the instantaneous rate of fishing mortality and Y the yield or catch is also considered. Below are the models under this group

a. *Swept area method*

Let a be the area swept by one unit of effort; A - the total area inhabited by the stock in question; X-the escapement factor, in otherwords the fraction of fish caught by the unit effort, and c-the catch per unit effort during the period of survey. Then, the stock size B, is given by

$$B = \frac{c.\ A}{X.\ a} \qquad \ldots(2)$$

The area a swept by one unit of effort in case of trawling is given by

$$a = t.v.w \qquad \ldots(3)$$

where t is the time spent for trawling, v the velocity of the craft and w the width of the area swept by the gear. It is clear from the above that the expected yield for the gears from the entire area is

$$Y = FB = XB = \frac{c.\ A}{a} \qquad \ldots(4)$$

b. *Biomass approach*

Using instantaneous fishing (F), natural (M) mortality rates and intrinsic rate of increase (r_m) of a population

$$B = Y/F \qquad \ldots(5)$$

obtainable from (3) above and

$$Msy = r_m.\ B\infty/4 \qquad \ldots(6)$$

$$= 0.5\ M.B_v \qquad \ldots(7)$$

where $B\infty$ indicates the carrying capacity of the water body and B_v the virgin biomass (Gulland, 1971).

c. *Surplus production model*

Removal from a stock through catches reduces the biomass of the stock. The rate of this removal may be greater or equal or less than that of addition in the stock through growth and recruitment. In the first case the biomass will decrease; in the second it remains constant in which case the corresponding catch is called the equilibrium catch Y_e for that level of biomass and in the last case the biomass will increase. In general when a stock is exploited the change in its biomass depends on its intrinsic rates of natural growth and the catch. This can be expressed as

$$\frac{1}{\bar{B}} \frac{(\triangle B)}{\triangle t} = f(\bar{B}) - F \qquad \ldots(8)$$

where $\bar{B}$ is the mean biomass during the time interval $\triangle t$ and $f(\bar{B})$ the rate of natural growth and F the rate of catch removal. Hence

$$\triangle\bar{B} = \bar{B}\ \triangle t\ (f(\bar{B}) - F) \qquad \ldots(9)$$

Under equilibrium condition $\triangle \bar{B} = 0$. Hence, the condition for equilibrium is

$$f(\bar{B}) = F \qquad \ldots(10)$$

Moreover, the equilibrium catch Y_e is, using (10)

$$Y_e = F\bar{B}\,\Delta t = \bar{B}\, f(\bar{B})\,\Delta t \qquad \ldots(11)$$

c 1. *The Schaefer model*

The intrinsic rate of natural growth depends on the biomass. It increases or decreases as the biomass increases or decreases. As a first approximation let the relation be linear. When a stock is exploited its biomass decreases. Hence, according to Schaefer (1954)

$$f(\bar{B}) = m - K\bar{B} \qquad \ldots(12)$$

When $\bar{B}$ has reached its maximum, B_∞, then $f(\bar{B}) = O$. Hence,

$$m = KB_\infty \quad \text{and} \quad f(\bar{B}) = K(B_\infty - \bar{B}).$$

Under equilibrium condition $f(\bar{B}) = F$ and $Y_e = F\bar{B}$. Hence

$$Y_e = K\bar{B}(B_\infty - \bar{B}). \qquad \ldots(13)$$

Equation (13) leads to the famous logistic growth curve

where $\frac{dB_t}{dt} = kB(B_\infty - \bar{B}) / B_\infty$,

$K = kB_\infty$ leading to

$$B_t = B_\infty / \left\{1 + \exp[-k(t-t_0)]\right\} \quad \ldots \quad (14)$$

From the above model one can obtain MSY (Y max) and the effort (F max) required to obtain this MSY, by equating $dY_e / dB = O$. The equations satisfying the equilibrium conditions are

$$\left.\begin{aligned} \bar{B} &= B_\infty - \frac{F}{K} \\ Y_e &= F\left(B_\infty - \frac{F}{K}\right) \\ &= K\bar{B}\,(B_\infty - \bar{B}) \end{aligned}\right\} \qquad \ldots(15)$$

Hence,

$$\left.\begin{aligned} F\max &= KB_\infty/2 \\ B\max &= B_\infty/2 \\ \text{and } Y\max &= KB_\infty^2/4 \end{aligned}\right\} \qquad \ldots(16)$$

c. 2. *Exponential model*

In the place of linear relationship assumed above between f (B) and B, data in practice have indicated non-linear relationship. Hence, (Garrod, 1969; Fox 1970) an exponential relationship is suggested such that

$$f(\bar{B}) = m - K \ln \bar{B} \qquad \ldots(17)$$

In this case the equations under equilibrium condition are

$$\left.\begin{aligned} \bar{B} &= B_\infty \exp(-F/K) \\ Y_e &= FB_\infty \exp(-F/K) \\ \text{and } Y_e &= K\bar{B} \ln(B_\infty/\bar{B}) \end{aligned}\right\} \qquad \ldots(18)$$

leading to

$F\max = K$; $B\max = B_\infty/e$ and

$Y\max = KB_\infty/e$.

c. 3. *Pella and Tomlinson model*

Generalising the above assumption Pella and Tomilinson (1969) proposed

$$f(B) = K(B_\infty^{m-1} - \bar{B}^{m-1}) \qquad \ldots(19)$$

The equations under equilibrium conditions are

$$\left.\begin{aligned} \bar{B} &= \left(B_\infty^{m-1} - \frac{F}{K}\right)\frac{1}{m-1} \\ Y_e &= F\left(B_\infty^{m-1} - \frac{F}{K}\right)\frac{1}{m-1} \\ \text{and } Y_e &= K\bar{B}\,(B_\infty^{m-1} - \bar{B}^{m-1}) \end{aligned}\right\} \qquad \ldots(20)$$

leading to

$$F\max = K\frac{m-1}{m}B_\infty^{m-1};\ B\max B = \infty \left(\frac{1}{m}\right)^{\frac{1}{m-1}}$$

$$\text{and} \quad Y\max = K\frac{m-1}{m}B_\infty^{m}\left(\frac{1}{m}\right)^{\frac{1}{m-1}}$$

d. *Successive removal methods*

Under this group, it is assumed that the change in the stocks is only due to catch removals and during fishing no other change takes place.

d. 1. Let p be the probability of capture, c_i the i^{th} catch ($i = 1, 2, \ldots\ldots, k$), N the stock number then the probability of getting k successive catches given p and N is denoted by

$$f(C_1, C_2, \ldots C_k/N, p) = \prod_{i=1}^{k} \binom{N - \sum_{j=0}^{i-1} Cj}{c_i} p^{ci} q^{di} \qquad \ldots(21)$$

where $q = 1\text{-}p$ and $d_i = N - \sum_{j=0}^{i} c_j$

The likelihood function is given by

$$\log L(N, p) = \ln NT - \ln [(N\text{-}T)\,!] + T \ln p + (kN\text{-}X\text{-}T) \ln(1\text{-}p) - \sum_{i=1}^{k} \ln (c_i\,!) \qquad \ldots(22)$$

where $T = \sum_{i=1}^{k} c_j$

The likelihood equation for p is

$$\hat{p} = \frac{T}{kN\text{-}X} \left(\text{or } \hat{N} = \frac{X}{k} + \frac{T}{pk}\right) \qquad \ldots(23)$$

When $k = 2$

$$\hat{p} = \frac{C_1 + C_2}{2N - C_1} \left(\text{or } \hat{N} = \frac{pc_1 + c_1 + c_2}{2p}\right) \qquad \ldots(24)$$

The likelihood equation for N is complicated due to the presence of factorial terms. Using stirling formula

$$O = \frac{-T}{2N(N-T)} + k \ln\left[1 - \frac{T}{kN\text{-}X}\right] - \ln\left(1 - \frac{T}{N}\right) \qquad \ldots(25)$$

the first log term is relatively small when N-T and T are large since the log term taken together is $O(T^2/N^2)$ and the solution leads to a polynomial equation of degree k-1 in N after neglecting this term (Harding *et al.*, 1984). When $k = 2$ we get

$$\hat{N} = \frac{C_1^2}{C_1 - C_2} \quad \text{and} \quad \hat{p} = \frac{C_1 - C_2}{C_1} \qquad \ldots(26)$$

d. 2. *Leslie method*

Assuming catch per unit of effort is an index of the stock abundance, Leslie (1952) proposes

$$\frac{C_t}{f_t} = q \,.\, N_t \qquad \ldots(27)$$

Where C_t is the catch, f_t the fishing effort, N_t the mean population size during 't' and q the catchability coefficient. Now

$$N_t = N_0 - K_t \qquad \ldots(28)$$

Where N_0 is the initial size of the stock and K_t the cumulative catch to the start of interval 't' plus half of that taken during the interval. Hence

$$\frac{C_t}{f_t} = q\, N_0 - q\, K_t \qquad \ldots(29)$$

which is linear in K_t and C_t/f_t. Thus the parameters q and N_0 can be estimated from (29).

d. 3. *De Lury method*

A slight modification in the Leslie method, considered above, leads to that of De Lury (1951). Taking

$$\frac{C_t}{f_t} = q\, No. \frac{N_t}{N_0} \qquad \ldots(30)$$

and assuming that the fraction of stock taken by a unit of effort is small, for example 0.02 or less - we have,

$$N_t = N_0 \exp(-q E_t) \qquad \ldots(31)$$

Where E_t is the cumulative fishing effort upto the start of the interval t plus half that during that interval. From (30) and (31) and taking logarithms, we get

$$\begin{aligned} \ln(C_t/f_t) &= \ln(q N_0) + \ln(N_t/N_0) \\ &= \ln(q N_0) - q E_t \end{aligned} \qquad (32)$$

Thus (32) is linear in cumulative effort E_t and log (C_t/ft) and q and N_0 can be estimated in the usual way.

As long as there is no error in K_t, Leslie method provides unbiased estimates of q and N_0. In the case of De Lury method effective fishing effort tends to be less accurate than catch statistics and since the relative errors of log (C_t / f_t) and E_t will usually be unknown, it may not be possible to obtain unbiased estimates of q and N_0. Hence, Leslie method is generally preferable.

d. 4. *Ricker method*

$$N_0 - K_t = N_0 \, (1-q)^{E_t} \qquad \ldots(33)$$

in the above notations. Hence from (29)

$$\frac{C_t}{f_t} = q\,(N_0 - K_t) = q\,N_0\,(1\text{-}q)^{E_t} \qquad \ldots(34)$$

Taking logarithms, this leads to

$$\log(C_t/f_t) = \log(q\,N_0) + E_t \log(1\text{-}q) \quad \ldots(35)$$

e. *Capture-recapture methods*

Starting with simple hypergeometric model in single release of marked ones there are methods covering multiple release and recapture systems both in closed and open populations. A wealth of material and references are available in Seber (1973). Here we shall see some of the important models for assessment of stocks in closed populations. The basic assumptions involved in obtaining valid estimates of N, the stock size, are:

i. The population is closed and hence N is constant.

ii. All animals whether marked or not have the same probability of being caught and

iii. There is no loss in marks during the interval between the sampling periods and at the reporting stage.

A simple case in single mark release is the famous Petersen estimate. We shall consider it here along with improved estimates suggested by chapman (1951) and Robson and Regier (1964). Under the above assumptions suppose n_1 animals are taken from a population of N animals, marked and released. Then a second sample of n_2 animals are taken after allowing sufficient time for marked animals to mix with the rest. Suppose m_2 animals are found marked in the second sample. Then Peterson estimate is

$$\frac{m_2}{n_1} = \frac{n_2}{N} \text{ or } \hat{N} = \frac{n_1\,n_2}{m_2}$$

This estimate is obtainable from the following models.

e. 1. *Hypergeometric model*

Where the conditional distribution of m_2 given n_1 and n_2 is

$$f(m_2/n_1, n_2) = \binom{n_1}{m_2}\binom{N-n_1}{n_2-m_2} \Big/ \binom{N}{n_2} \qquad \ldots(36)$$

Accordingly N is a best asymptotically normal estimate of N as $N \to \infty$. However, it is biased and the bias is large for small samples. Chapman (1951) suggests an unbiased estimate when

$n_1 + n_2 \geq N$ where,

$$N^* = \frac{(n_1 + 1)\ (n_2 + 1)}{(m_2 + 1)} - 1 \qquad \ldots(37)$$

When $n_1 + n_2 < N$, Robson and Regier (1964) indicate that the expected value of N^*

$$E\ [N^*/n_1, n_2] = N - Nb$$

and the bias element b may be made as small as possible where

$b = \exp\ [-\ (n_1+1)\ \ (n_2+1)/N]$ and

when the recaptures, $m_2 \geq 7$ one be sure that the bias in N^* is negligible with 95 per cent confidence.

e. 2. *Bailey-inverse sampling-method*

In this case n_1 and m_2 are fixed parameters and n_2 is a random variable. In otherwords sampling is continued till m_2 marked ones are recaptured. Bailey (1951) suggests the negative hypergeometric distribution for n_2 given

n_1 and m_2 such that

$f\,(n_2/n_1, m_2) =$

$$\binom{n_1}{m_2-1}\binom{N-n_1}{n_2-m_2}\left(\frac{n_1-m_2+1}{N-n_2+1}\right)\Big/\binom{N}{n_2-1} \qquad \ldots(38)$$

The modified maximum likelihood estimate (m.l.e) is

$$\hat{N} = [n_2(n_1+1)/m_2] - 1 \qquad \ldots(39)$$

Which is unbiased and has exact variance.

A simple extension of Peterson method is to a series samples of sizes n_i, $i = 1, 2, \ldots\ldots, s$. (Schnabel, 1938). For this purpose, in the notation of Seber (1973), let N be the stock size, s the number of samples, n_i the size of i^{th} sample and m_i number of marked ones in n_i

$(i = 1, 2, \ldots\ldots s)$, $u_i = n_i - m_i$ and

$$M_i = \sum_{j=1}^{i-1} u_j \quad (i = 1, 2, \ldots\ldots, s + 1)$$

the number of unmarked individuals in the stock just before the i^{th} sample is taken. Evidently $m_1 = M_1 = 0$ and $M_2 = u_1 = n_1$ as there are no marked ones in the first sample. M_{s+1} indicates the total number of different animals caught throughout the experiment.

e. 3. *The generalised hypergeometric model*

Let a_ω be the number of animals having the same capture history ω such that ω is an non-empty subset of the integers $[1, 2, \ldots\ s]$, a_{124} representing those animals caught in the first, second and fourth samples only and $r = \sum_\omega a_\omega$. Let p_ω be the probability of an animal caught having history w which is the same for each animal and each animal acts independently then the joint distribution (Seber, 1973) of the random variables (r.v) a_ω is given by

$$f\ [a_\omega] = N!\,Q^{N-r} \prod_\omega P_\omega{}^{a_\omega} \Big/ \prod_\omega a_\omega!\ (N-r)! \qquad \ldots(40)$$

Where $Q = 1 - \sum_\omega p_\omega$

Darroch (1958) shows that

$$f\,([a_\omega]) = N! \prod_{i=1}^{s} p_i{}^{n_i}\ q_i{}^{N-n_i} \Big/ \prod_\omega a_\omega!\ (N-r)! \qquad \ldots(41)$$

where p_i is the probability of each animal being caught in the i^{th} sample and the events in the i^{th} sample $(i = 1, 2, \ldots\ldots, s)$ are independent. Since in this case

$Q = \prod_{i=1}^{s} q_i$ and $p_{124} = p_1 p_2 q_3 p_4 \cdots q_s =$

$p_1\, p_2\, p_4\, Q\ /\ q_1\, q_2\, q_3$ etc.

From the above the $[n_i]$ are independent binomial variables so that

$$f([ni]) = \prod_{i=1}^{s} \binom{N}{ni} p_i^{ni} q_i^{N-ni} \qquad \ldots(42)$$

Hence

$$f([a_\omega]/[ni]) = N! \prod_{i=1}^{s} \binom{N}{n_i}^{-1} \Big/ \prod_{\omega} a_\omega! \, (N-r)! \qquad \ldots(43)$$

The m.l.e. of N is given by

$$(1-r/N) = \prod_{i=1}^{s} (1-n_i/N) \qquad \ldots \ (44)$$

Case 1. $s = 2$

$N = n_1 \, n_2/m_2$ (Petersen estimate)

Case 2. $s = 3$

$N^2 (m_2+m_3) - N (n_1 n_2 + n_2 n_3 + n_1 n_3) + n_1 n_2 n_3 = 0$

is a quadratic in N which can be solved.

Case 3. $s > 3$ Iterative methods are required for obtaining estimates for N. However, Robson and Regier (1964) and Chapman (1952) have indicated methods to obtain estimates for N.

e. 4. *Inverse Schnabel census*

Extending this to inverse sampling scheme one gets

$$f\left(\underset{i=3,4,\ldots,s}{\{M_i\}}, \ r, \underset{i=2,3,\ldots,s}{\{n_i\}} \Big/ \ N,s,n_1, \underset{i=2,3,\ldots,s}{\{m_i\}} \right)$$

$$= \prod_{i=2}^{s} \left\{ \binom{m_i}{m_i-1} \binom{N-m_i}{u_i} \left(\frac{M_i-m_i+1}{N-n_i+1} \right) \Big/ \binom{N}{n_i-1} \right\} \qquad \ldots \ (45)$$

M.l.e.$\hat{N}$ is obtainable as in (44), but its asymptotic bias and variance are not easily available. Hence Chapman (1952) suggests an unbiased estimate

$$\hat{N} = \sum_{i=2}^{s} \left(\frac{n_i (M_i+1)}{m_i} - 1 \right) \Big/ (s-1) \ \ldots \ (46)$$

It may be noted that in the inverse sampling scheme considered in e. 2 and e. 4 Chapman (1952) has indicated that considering u_i the number of animals not marked in the ith sample as a random variable in the place of m_i, the marked ones, would improve the precision of the stock estimates. Extensive work on the above lines for open populations has been done and the details are available in Seber (1973). All these estimates do have their variance estimates either accurate wherever possible or approximate in other cases.

f. *Relative response model*

This model depends on successive catches to predict the maximum catch that the fishery can sustain. There are three assumptions (Alagaraja, 1984) for success of this model. These are (1) Stocks existing in a particular area are exploited by various types of gear that are not species specific. This implies that the effect of fishing a mixture of stocks by these gears is proportional to the relative abundance of stocks in the mixture. (2) The fishing is increased over a period of time till the optimum level is achieved. (3) When the effort is increased the catches also increase till a maximum level is reached, but the rate of increase increases first then decreases and finally reaches to nil. In the progressive fisheries where multispecies are exploited by multigears and where evaluation of effective effort poses problems particularly in tropical fisheries, this model is useful. The model is

$$C_t - C_{t-1} = f(C_{t-1}) \qquad \ldots \ (47)$$

A simple version of the above is a linear relationship between the successive catches, namely

$$C_{t+1} = a + b \, C_t \qquad \ldots \ (48)$$

In the progressive fishery the level of maximum catch can be predicted and suitable management measures could be suggested in advance to get sustainable yield from the fishery. (48) is of the same form of the well known equation in von Bertalanffy's growth model.

Hence

$$C_{t+1} = C_{max} (1 - e^{-k}) + C_t e^{-k} \quad \dots (49)$$

and

$$C_{max} = a/(1-b) \quad \dots (50)$$

in the notation of (48)

g. *Quick estimates*

When the fisheries is in progress, in the absence of earlier history of the fisheries, it is worthwhile to have quick estimates based on known statistics. Timeliness of the estimates is so important to take decision on the level of exploitation, the necessity of which is felt by the fishery managers.

g. 1. *Comparison method*

On the basis of yield gradients based on catch estimates or primary productivity of a known area, production in other areas having similar characteristics can be estimated. To obtain the potential maximum yield of the African Coasts, the estimates available for comparable European Coast were used (Gulland, 1971). Using available data on production of rivers (Y) and the area of their main course (X) (Welcome, 1976 in Troadec, 1976) it is suggested that the relationship

$$Y = a X^b \quad \dots (51)$$

can be used for similar river systems.

g. 2. *Indicator method*

If an indicator on potential yield that is easily and quickly measureable is available then that indicator could be profitably utilised for assessing yield. Morphoedaphic index (M), equal to the ratio of total weight of the dissolved solid matter to average depth of the water body is useful in this regard. The well known relationship is

$$Y = a M^b \quad \dots (52)$$

g. 3. *Productivity approach*

Knowledge on production at successive trophic levels is required in this approach. Several attempts have been made on these lines to assess potential yield (Paulik, 1971). Owing to the complexity of trophic relationships the results obtained by different authors vary widely, thus casting doubts on this approach. Cushing (1969) indicates the difficulties involved in this approach. Interested readers may refer to these references for further information.

MICRO-ANALYTIC MODELS

In contrast to the macro-analytic models considered earlier, micro-analytic models or otherwise called dynamic pool models (Clark, 1976) take into account recruitment, mortality, age, growth and other factors affecting a stock. The models that are considered here are based on two major assumptions namely the stock under study is in a steady (or equilibrium) state. In otherwords recruitment, growth and mortality are constant. This results in an annual yield from the entire stock equivalent to the yield from a cohort during its entire life span. Secondly, the yield is directly related to the recruitment. Under these assumptions it is clear that yield-per-recruit (Y/R) is an index of the stock and attempts are made to estimate Y/R to study the condition of the stocks exploited. The most widely used model is the Beverton and Holt model (Beverton and Holt, 1957).

a. 1. *Beverton and Holt model*

Let N_t be the number of fish alive at age t; M and F the instantaneous rates of natural and fishing mortalities respectively and W_t, the average weight of a fish at age 't'. Then in the interval t, $t + \Delta t$ the numbers ΔC_t and weight ΔY_t, which are caught, are given by $\Delta C_t = F N_t \Delta t$ and $\Delta Y_t = F N_t W_t \Delta t$ Suppose t_r is the age at recruitment and t_l the total life expected, then catches during this life span are denoted by

$$c = \int_{t_r}^{t_l} dc_t = \int_{t_r}^{t_l} F N_t \; dt$$

and

$$y = \int_{t_r}^{t_l} dy_t = \int_{t_r}^{t_l} F N_t \; W_t \; dt$$

Though fish is available in the fishing ground at the age t_r its size at first capture depends on the gear employed. Suppose the age at first capture be t_c then from t_r to t_c no fishing mortality takes place. Only from t_c onwards fishing mortality along with natural mortality takes place on the stock. Hence we have $F = O$ and $Z = M$ when $t \leq t_c$. for $Z = F + M$ and $Z = F + M$ when $t > t_c$. Let R be the recruits at t_r.

Then $N_t = R \exp\,[-M(t - t_r)]$ for $t \leq t_c$. If R' are the number of fish at $t = t_c$ then

$N_t = R' \exp\,[-Z(t - t_r)]$ for $t > t_c$

where $R' = R \exp\,[-M(t_c - t_r)]$. In these notations the number of fish caught is

$$c = \int_{t_c}^{t_L} R' F \exp\,[-Z(t - t_c)]\, dt$$

$$= R' \tfrac{F}{Z} \left(1 - \exp\,[-Z(t_l - t_c)]\right)$$

$$= R \tfrac{F}{Z} \exp\,[-M(t_c - t_r)] \;\{1 - \exp\,[-Z(t_L - t_c)]\} \quad \ldots (53)$$

Case 1. When t_L is sufficiently large, then the last term becomes negligible and

$$C = R \tfrac{F}{Z} \exp\,[-M\;(t_c - t_r)]$$
$$= \tfrac{F}{Z}\; R' \quad \ldots (54)$$

The catch in terms of weight can be obtained once time-weight relationship is known. For this purpose von Bertalanffy's growth equation

$$W_t = W_\infty \left[1 - e^{-k(t - t_o)}\right]^3 \quad \ldots (55)$$

is used. This may be re-written as

$$W_t = W_\infty \sum_{n=o}^{3} U_n \exp\,[-nk\,(t - t_o)]$$

Hence

$$Y = \int_{t_c}^{t_L} FR' W_\infty \exp\,[-Z\,(t - t_c) \sum_{n=o}^{3} U_n$$
$$\exp\,[-nk\,(t - t_o)]\, dt$$

On integrating we get

$$Y = FR' W_\infty \sum_{o}^{3} \frac{U_n}{Z + nk} \exp\,[-nk\,(t_c - t_o)] x$$
$$\left\{1 - \exp\,[-(Z + nk)\,(t_L - t_c)]\right\} \quad \ldots (56)$$

where $U_O = 1$, $U_1 = -3$, $U_2 = 3$ and $U_3 = -1$

Case 1. When t_L is sufficiently large, the last term becomes negligible and we have

$$Y = FR' W_\infty \sum_{o}^{3} U_n \exp\,[-nk\,(t_c - t_o)] / (z + nk)$$

$$= FR \exp\,[-M\,(t_c - t_r)]\; W_\infty \sum_{o}^{3} U_n \exp$$
$$[-nk\,(t_c - t_o)] / (z + nk)$$

The yield-per-recruit thus becomes

$$Y/R = F \exp\,[-M\,(t_c - t_r)]\; W_\infty \sum_{o}^{3} U_n \exp$$
$$[-nk\,(t_c - t_o)] / (z + nk) \quad \ldots (57)$$

Since this a function of F, t_c and Y/R, the effect of gear both in terms of intensity (F) and the size at first capture (t_c) on Y/R could be studied. Accordingly useful suggestions on the effort imposed on fishery can be given. This is straight forward and elegant. Yield Tables are available (Beverton and Holt, 1964) to draw yield isopleths for drawing conclusions from the nature of existing fishery.

a. 2. *Jones method*

In (55) isometric growth is assumed and hence cubic law is suggested for growth in weight. In general assuming allometric growth (55) can be written as

$$W_t = W_\infty \; [1 - \exp\{-k(t - t_0)\}]^b \quad \ldots \quad (58)$$

In this case yield-per recruit integral becomes an incomplete beta function of the form

$$Y/R = (F/k)\exp[-M(t_c - t_r) + z\,(t_c - t_0]$$

$$W_\infty \int_0^{z} x^{p-1}\,(1-x)^{qp-1}\,dx \quad \ldots \quad (59)$$

Where $p = z/k$, $q = b + 1$ and

$$z = \exp\left\{-k\left(t_c - t_0\right)\right\}$$

and the values are tabulated (Wilimovsky and Wicklund, 1963).

a. 3. *Ricker model*

Ricker (1975) proposes a simpler method with no assumption on the form of growth. Using $Y = FB$, he suggests.

$$Y_E = \sum_{t=t_c}^{t_L} F_t\,\bar{B}_t \quad \ldots \quad (60)$$

As a first approximation $\bar{B}_t$ is the average of initial and final biomasses indicated by $\bar{B}_t = (B_t + B_{t+1})/2 = B_t[1 + \exp(G_t - Z_t)]/2$ where G_t and Z_t are the instantaneous growth and mortality rates respectively. Hence the yield equation becomes

$$y_E = \sum_{t=t_r}^{t_L} F_t B_t [1 + \exp(G_t - Z_t)]/2$$

If a stock were to increase or decrease exponentially then

$$\bar{B}_t = B_t[\exp(G_t - Z_t) - 1]/(G_t - Z_t) \quad \ldots \quad (61)$$

This can be substituted in (60) and obtain the corresponding yield equation.

a. 4. *Cohort analysis*

Demographic composition of a cohort with the corresponding rates of mortalities determines more precisely the condition of a stock. Catches at regular intervals (annual) taken from a single cohort are useful to estimate the abundance of the stock and the fishing mortality. In this type of analysis the estimates of stock sizes at each age/size are estimated backwards finally arriving at the initial stock size. For this purpose the instantaneous rates of natural (M) and fishing (F) mortalities are assumed to be known. Let N_i be the number of fish at the start of i^{th} period; S_i the rate of survival during i; D_i the number die during i and C_i the catch during i. Hence we have

$$N_{i+1} = N_i S_i\,;\; D_i = N_{i-1}\,;\; S_i = \exp(-Z_i)$$

where $Z_i = F_i + M_i$; $D_i = Z_i\,N$ and $C_i = F_i\,\bar{N}_i$ using $C = (F_i / Z_i)\,N_i\,[1 - \exp(-Z_i)]$

$$(62)$$

and the above relation leading to $N_i = D_i/(1 - S_i)$ where $D_i = Z_i\,\bar{N}_i$ and $\bar{N}_i = C_i/F_i$, from the initial values of C_i, M_i and F_i successive stock abundances can easily be estimated. Slight modifications in this approach is suggested by Pope (1972) otherwise called virtual population analysis (VPA).

FISH PRODUCTION EVALUATION

Evaluation of fish production from cultured waters is very important to assess the carrying capacity of a water body. Using biomass $B_t = N_t\,W$ in the usual notation production in a given time interval is defined as (Ivlev, 1966) the total elaboration of animal tissue during that time interval, including what is formed by individuals that do not survive till the end of that time interval. Ricker (1946) and Allen (1950) propose

$$P_1 = G\bar{B} \qquad \ldots(63)$$

where P is the production during the time interval, G the instantaneous growth rate, $\bar{B}$ the average biomass and

$$\int_0^t dp_r = \int_0^t N_r \, d\bar{w}_r \qquad \ldots(64)$$

for (63) it is assumed that growth follows exponential law namely $W_t = W_0 \, e^{Gt}$ and change in population number, N_r, may have any form. Alagaraja (1980) suggests

$$P = Y + ZB \qquad \ldots(65)$$

$$\text{and} = b\,\bar{N} \qquad \ldots(66)$$

where Y is the net yield and b is the slope as defined below. In (65) it is assumed that change in population numbers follows exponential law namely $N_t = N_0 \, e^{-Zt}$ and the growth may have any form. In (66) both growth and numbers are linear with time t such that

$W_t = a + bt$ and $N_t = a_1 + b_1 \, t$. When intervals of sampling is small these linear assumption will give estimates closer to the actual values. In culture experiments intervals of sampling are always small and hence the simple form (66) can successfully be used.

STOCHASTIC MODELS

In the exponential growth model we have from the Poison Process

$$E\,(N_t) = N_0 \, e^{-Zt} \qquad \ldots(67)$$

leading to

$$E\,(N_{t+1}) = N_t \, e^{-Z} \qquad \ldots(68)$$

and

$$\mathrm{Var}\,(N_{t+1}) = \frac{b+d}{b-d} \left[\exp 2\,(b-d) - \exp\,(b-d) \right] N_t \qquad \ldots(69)$$

Using Monte Carlo methods, once N_t is known N_{t+1} can be found out. Here b and d indicate intrinsic rate of birth and death rates.

Bartlet (1960) extended the logistic growth to take care of chance effect and the effect of density of birth and death rates separately. He shows that the probability Pr of Population size N becoming N + 1 is

$$P_r\,(N \rightarrow N+1) = [\,bN - \infty_1 N^2\,] / \; (b+d)\,N - (\infty_1 - \infty_2)\,N^2\,]$$

and similarly

$$P_t\,(N \rightarrow N-1) = [\,dN + \propto_2 N^2\,] \; [\,(b+d)\,N - (\propto_1 - \propto_2)\,N^2\,] / \qquad \ldots(70)$$

Where $\propto_1$ and $\propto_2$ are the effects of density on birth rate and death rate respectiviely. Here also using Monte Carlo methods we can find the estimates for N_{t+1} once N_t is known.

Riffenburgh (1969) indicates use of non stationary Markov Chains in his interpopulation dynamics model to analyse and model the passage of energy through an ecological system composed of the three fisheries of sardines, anchovies and hake. He also mentions the problems encountered in obtaining solutions due to weakness of data base. As mentioned earlier much work is yet to be done on stochastic approach lines in fish stock assessments.

GENERAL MULTISPECIES APPROACH

Generalising Schaefer model to multispecies we have

$$f_i(N_1, N_2, \ldots\ldots N_m) = \frac{1}{N_i}\frac{dN_i}{dt} \quad \ldots(71)$$

The simplest form of (71) is one where inter specific interaction is absent. Hence

$$f_i(N_1, N_2, \ldots, N_m) = \alpha_i + \beta_i N_i \quad \ldots (72)$$

However, there are complex ecosystem models (Andersen and Ursin, 1978). Here we shall consider O model generalising that of Schaefer. In the above forms this model is

$$\frac{1}{N}\frac{dN}{dt} = b - aN\text{-}F$$

When $F = o$, virgin population equilibrium is at $N_0 = b/a$ and when $F = qf$ the sustainable yield $Y(f) = qf(b\text{-}qf)/a$. Thus the sustainable yield curve as a function of f is a parabola with a maximum at ½ No. Extending this to m species

$$\frac{1}{N_i}\frac{dNi}{dt} = b_i - \sum_{j=1}^{m} a_{ij} N_j - q_i f \quad \ldots (73)$$

$i = 1, 2, \ldots, m$. If $a_{ij} = o$ for $i \neq j$, then (73) reduces to m independent single species Schaefer model. In general (73) covers Lotka-Volterra form with fishing effect included. Pope (1976, 1979) obtains at $\left\{ N_i^* \right\}$., $i = 1, 2 \ldots m$ when the system is at equilibrium.

$$B - A\underset{\sim}{N}^* - Qf = o \quad \ldots(74)$$

where B, $\underset{\sim}{N}^*$ and Q are (1 x m) matrices and A is an (m x m) matrix. Hence we have

$\underset{\sim}{N}^* = A^{-1}(B - Qf)$ and the total yield from the system at $\underline{N}^*$ is $Y = fQ^T\underline{N}^*$. This indicates that for any matrix A, Y is a parabolic function in f when Q is constant and none of $N^*$is zero. From the above.

$$Y = (\underset{\sim}{N}^{*T} B - \underset{\sim}{N}^{*T} A \underset{\sim}{N}^*) \quad \ldots (75)$$

Differentiating (75) with respect $\underset{\sim}{N}^*$ and equating to zero we get

$$\underset{\sim}{N}(MSY) = (A + A^T)^{-1} B \quad \ldots (76)$$

Hence when A is symmetric

$$\underline{N}(MSY) = A^{-1}B/2 = \underset{\sim}{N}o/2$$

Where $\underset{\sim}{N}o = A^{-1}B$ the virgin biomass. It may be noted that MSY for fixed Q will be less than overall MSY and the overall MSY is generally less than the sum of individual species MSYs. This approach is extended to other production models such as Fox and Pella and Tomlinson. For further details one may refer to Pope (1979).

To conclude it is difficult to pen down all the models used for fish stock assessment in a paper like this. However, it is felt that the increase in awarness among those who have mathematical background towards development of suitable models and the availability of computer facilities will go a very long way to bring out notable progress in fish stock assessments.

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