CHAPTER 18

# ROLE OF CALCULUS IN MARINE SCIENCES

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## 1. Introduction

Calculus has two parts: differential and integral calculus. Historically, differential calculus was concerned with finding lines tangent to curves and with calculating extrema (*i.e.*, maxima and minima) of curves. Integral calculus has its roots in attempting to determine the areas of regions bounded by curves or in finding the volumes of solids. The two parts of calculus are closely related: The basic operation of one can be considered the inverse of the other. This result is known as the fundamental theorem of calculus and goes back to Newton and Leibniz, who were the first to understand its meaning and to put it to use in solving difficult problems (Reference 3).

## 2. Worked out Examples

### 2.1 ) Differentiation

Differentiation is an aspect of calculus that enables us to determine how one quantity changes with regard to another. It tells you how quickly (or slowly) a function changes at a given point. Finding tangents, locating extrema, and calculating areas are basic geometric problems, and it may be somewhat surprising that their solution led to the development of methods that are useful in a wide range of scientific fields. The main reason for this historical development is that the slope of a tangent line at a given point is related to how quickly the function changes at that point. Knowing how quickly a function changes at a point opens up the possibility of a dynamic description of biology, such as a description of population growth, the speed at which a chemical reaction proceeds, the firing rate of neurons, and the speed at which an invasive species invades a new habitat. For this reason, calculus has been one of the most powerful tools in the mathematical formulation of scientific concepts (Reference 3).

By working out the following examples, you will be able to develop an appreciation of the usefulness of calculus across multiple fields.

**Example 1: Application of Differentiation** (References 1.a and 1.b)

Carlos has taken an initial dose of a prescription medication. The amount of medication, in milligrams, in Carlos's bloodstream after t hours is given by the following function:

$M(t) = 20 * e^{(-0.8*t)}$

What is the instantaneous rate of change of the remaining amount of medication after 1 hour? In what units is this rate of change measured?

Solution to Example 1

The instantaneous rate of change of M(t) is given by its derivative, M'(t). Therefore, the instantaneous rate of change of the remaining amount of medication after 1 hour is simply M'(1).

Therefore, we need to find the value of M'(t) at t=1 or

$M'(1)\ M'(t) = -16 * e^{(-0.8*t)}$

$M'(1) = -16* e^{(-0.8*1)}$

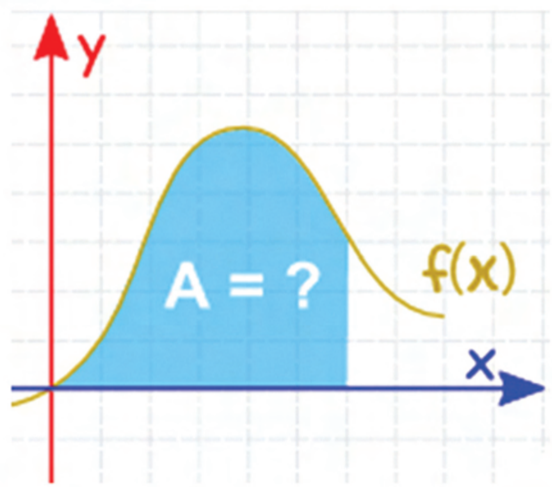
$M'(1) \sim 7.2$

M(t) is the amount of medication that remains in Carlos's blood stream after 't' hours. Therefore, the rate of change is measured in milligrams per hour.

In conclusion, the instantaneous rate of change of the remaining amount of medication after 1 hour is -7.2 milligrams per hour. The rate of change is negative because the amount of medication is decreasing.

**2.2) Integration** (References 1.a and 1.b)

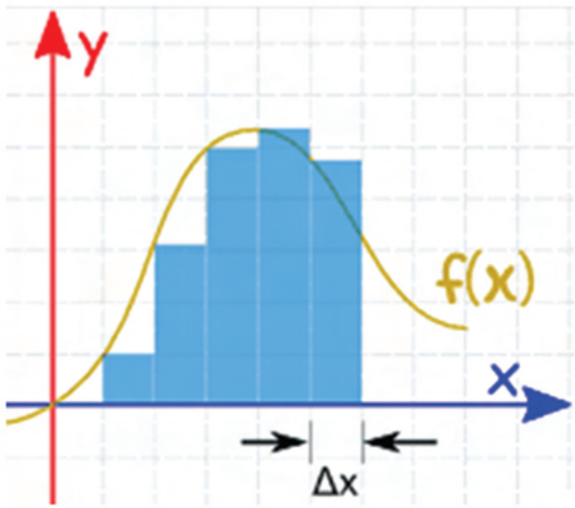
Integration is a way of adding slices, summing them to find the whole area of a curve. Integration can be used to find areas, volumes, central points and many useful things. But it is easier to start with finding the **area under the curve of a function** like this:



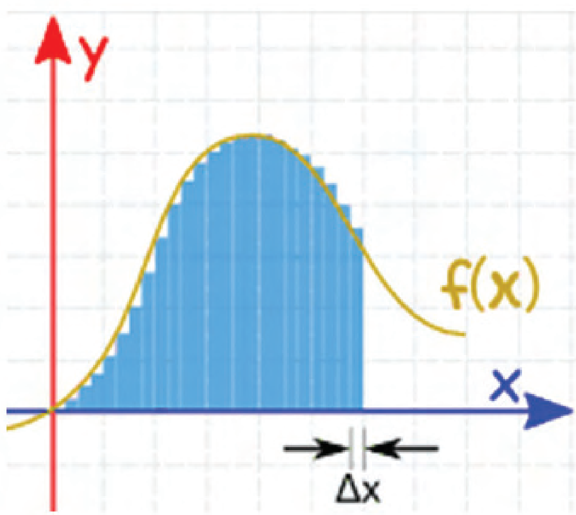
What is the area under **y = f(x)** ?

Slices

We could calculate the function at a few points and **add up slices of width Δx** like this (but the answer won't be very accurate):

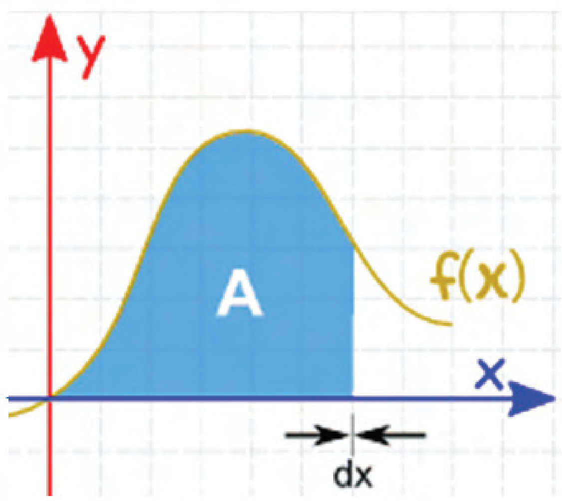



We can make **Δx** a lot smaller and **add up many small slices** (answer is getting better):

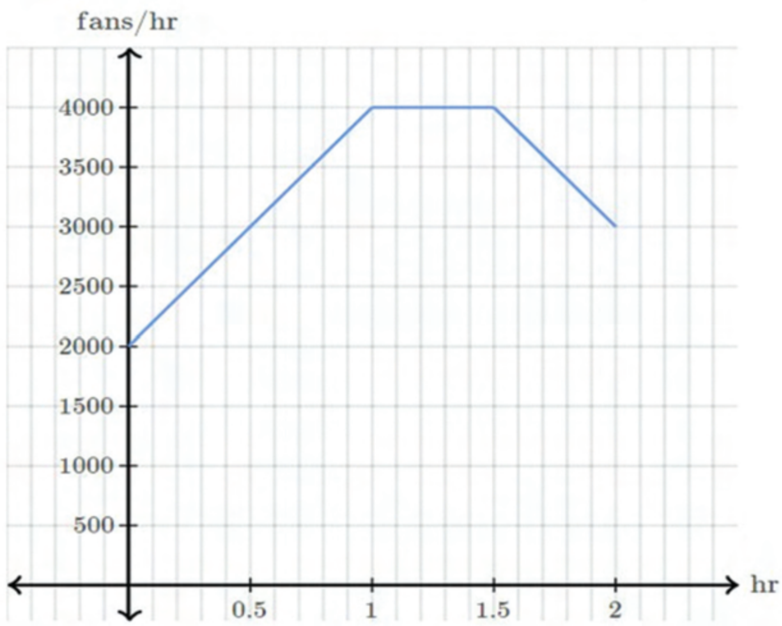


And as the slices **approach zero in width**, the answer approaches the **true answer**.

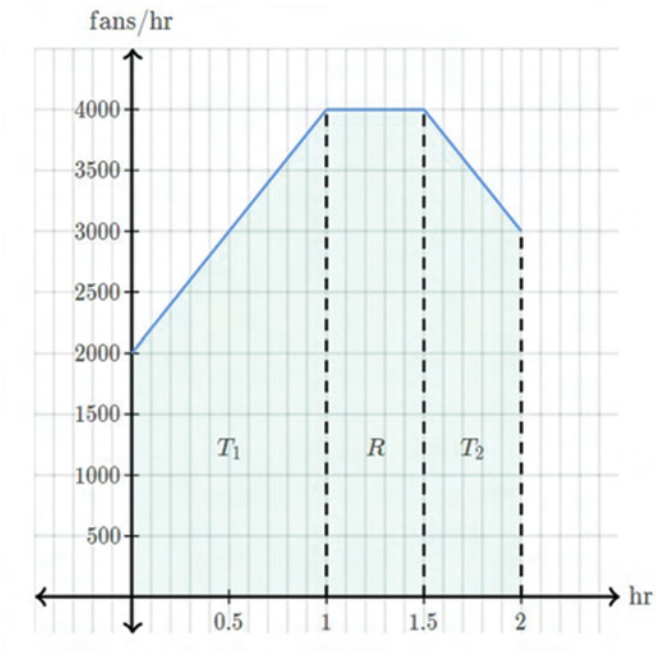
We now write **dx** to mean the **Δx** slices are approaching zero in width.




**Example 2: Application of Integration** (References 1.a and 1.b)



Solution to example 2:




**2.3) Application of Differential Equations** (Reference 2) Example 3: Given:

State variables : N - Amount of nutrient available

P - Phytoplankton

Process of interest - Photosynthetic production of organic matter

$$\frac{d}{dt} P = v_{max} f(N) P$$

where

$$f(N) = \frac{N}{kN+N}$$

When N is large

f(N) =1

When N is small

f(N)= $N/k_n$

*Case 1*

What would be the growth profile of Phytoplankton in presence of ample nutrients (*i.e.*, when N is large)? In such a scenario, keeping the plankton concentration a constant, how would Nutrient concentration vary with time?

*Case 2*

Keeping the plankton concentration a constant in presence of limited nutrients (*i.e.* when N is small), how will Nutrients change with respect to time?

**Solution to Example 3**

The nitrogen consumed by the phytoplankton for growth must be lost from the *Nutrients* state variable. Therefore,

$$\frac{d}{dt}P = v_{max}f(N)P$$

$$\frac{vd}{dt}N = v_{max}f(N)P$$

and

$$\frac{d}{dt}(P+N) = 0$$

Because the total *inventory* of nitrogen is conserved.

*Case 1*

When nutrient availability is ample, f(N) = 1

$$\frac{dP}{dt} v_{max}P \text{ ——— } (1)$$ After integrating with respect to time, we get

$$P = A\, e^{v_{max}t} \text{ ——— } (2)$$ **Growth of P will be exponential**

When plankton concentration is kept under ample nutrient conditions, we have

$$\frac{dN}{dt}P = v_{max}P \text{ ——— } (3)$$ After integration, we get

$N(t) = -v_{max}*P*t$ —————————— (4)

**N will decrease linearly with time as it is consumed to grow P**

*Case 2*

When nutrient availability is scarce, $f(N) = N/K_n$

Therefore,

$$\frac{dN}{dt} = \frac{v_{max}P}{kn} N \quad \text{——————————} \quad (5)$$

On integrating (5) with respect to time, we get

$$N = Ae^{\frac{v_{max}P}{kn}} \quad \text{——————————} \quad (6)$$

**N will exponentially decay to zero until it is exhausted**

## References


1) Websites:

   a) www.khanacademy.org

   b) https://www.mathsisfun.com/calculus/integration-introduction.html

2) Slides: Simple coupled physical-biogeochemical models of marine ecosystems - available at https://www.google.co.in/urlsa=t&rct=j&q=&esrc=s&source=web&cd=3&cad=rja&uact 8&ved=0ahUKEwiSgubj08LXAhVQtJQKHR0ZBY4QFggxMAI&url=https%3A%2F%2Fmarine.rutgers.edu%2Fdmcs%2Fms320%2F2016-09-21-Coupled-physical-biological-models.pptx&usg=AOvVaw0RID3Uk_WkhIvSPZpXmM28

3) Book: Calculus for Biology and Medicine authored by Claudia Neuhauser