# ESTIMATION OF LENGTH WEIGHT RELATIONSHIP IN FISHES

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Organisms generally increase in size (length, weight) during development. The key factors that influence the growth of fish are the quantity of food available, the number of fish utilizing same food source, temperature, oxygen and other water quality factors besides the size, age and sexual maturity of the fish. Every animal in its life exhibit growth both in length and in weight and the relationship between these two has both applied and basic importance. The length-weight relationship is one of the standard methods that yield authentic biological information and is of great importance in fishery assessments. It establishes the mathematical relationship between the two variables, length and weight, and helps in assessing the variations from the expected weight for the known length groups. This is particularly useful for computing the biomass of a sample of fish from the length-frequency of that sample. The parameter estimates of the relationship for a population of fish can be compared to average parameters for the region, parameter estimates from previous years, or parameter estimates among groups of fish to identify the relative condition or robustness of the population. Relationship between length and weight is required for setting up yield equation and sometimes it may be useful as a character to differentiate "small taxonomic units". It also helps in converting one variable into another. Of the two, length is easier to measure and can be converted into weight in which the catch is invariably expressed. The length weight relationship also provides means for finding out the "condition factor" and the seasonal changes in the condition factor are useful to determine the biological changes in the fish.

**Length-weight relationship -uses**

- to convert growth-in-length equations to growth-in-weight, for stock assessment models;
- for the estimation of the biomass of a species from length frequency distributions
- to calculate an estimate of the condition of fish; and
- for life history and morphological comparisons of life histories of a certain population from different regions

The relationship between weight (W) and length (L) in fishes has the form:

$W=aL^b$

Reprinted from the CMFRI, FRAD. 2014. Training Manual on Fish Stock Assessment and Management, p.150.

In this equation, the parameters ***a*** and ***b,*** usually termed as length weight parameters are to be estimated with the available length-weight data. Each species of fish will have a specific length-weight relationships or specific length - weight parameters. It may also differ between sexes and between stocks or those belonging to different geographical regions. The parameter ***a*** is a scaling coefficient for the weight at length of the fish species. The parameter ***b*** is a shape parameter for the body form of the fish species.

The length of a fish is often measured more accurately than the weight.

In theory, one might expect that the exponent ***b*** would have a value of roughly ***b*** = 3 because the volume of a 3-dimensional object is roughly proportional to the cube of length for a regularly shaped solid. Length is one dimensional whereas weight which depends on volume is three dimensional. Hence, there is thinking that weight of a fish is proportional to cube of the length of the fish. That is, there exists cubic relationship between weight and length of a fish. For an ideal fish which maintains the same shape b=3. Most species of fish do change their shape as they grow and so a cube relationship between length and weight would hardly be expected. It has also been found that while b may be different for fish from different localities, of different sexes, or for larval, immature and mature fish, it is often constant for fish similar in these respects. The length-weight relationship may thus be a character for the differentiation of small taxonomic units, like any other morphometric relationship. It may also change with metamorphosis or the onset of maturity.

In practice, fish that have thin elongated bodies will tend to have values of ***b*** that are less than 3 while fish that have thicker bodies will tend to have values of ***b*** that are greater than 3. Thus this also help to determine whether somatic growth is isometric (b=3) or allometric. Values of *b* smaller, equal and larger than 3 indicate isometry, negative allometry and positive allometry respectively. When $b>3$, large specimens increase in height or width faster than in length, either as the result of a change in body shape with size, or because the large specimens in the sample are in better condition than the small ones. Conversely, when $b<3$, either the large specimens have changed body shape, i.e., become more elongated, or the small specimens were in better nutritional condition at the time of sampling.

Thus the growth of fish length and weight is not proportionate or the relationship between length and weight is not linear. This means that when the length is increased the increase in weight is not proportionate to it. It is rather non-linear type of relationship. The estimation procedure for length – weight relationship is through linear regression. Since the above model of length-weight relationship is not linear it has to be transformed into linear type by applying logarithmic transformation.

If we take logarithm (*natural logarithm with base e*) the above model will become linear as

In $(W)$ = In $(a) + b$ In $(L)$ or $Y = A + b X$

where In($a$) is the intercept and ($b$) the slope or regression coefficient.

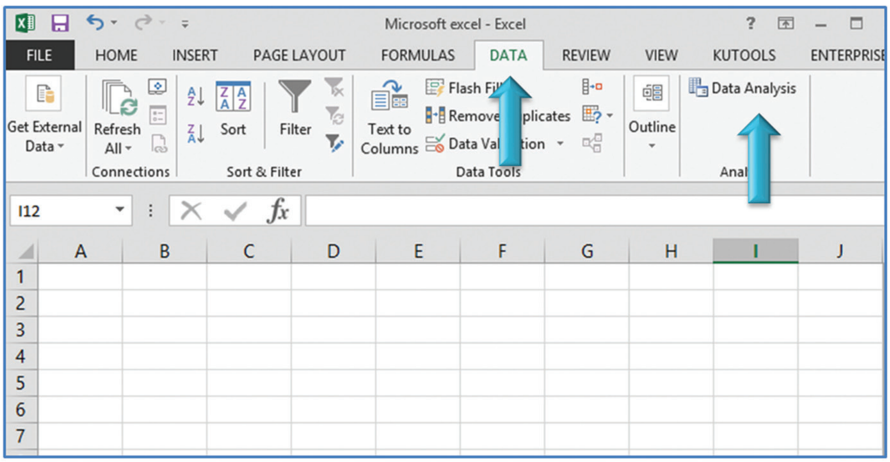
The above relationship is now linear and we can use the ordinary linear regression method for estimating the parameters of the relationship.

Data for fitting the length-weight relationship is collected randomly from the commercial catches and should represent fishes of all sizes, smallest to the biggest, and there should be enough samples for the analysis and estimation through regression. If our aim is to examine difference in length weight relationship between different sexes then data should be collected separately for males and females.

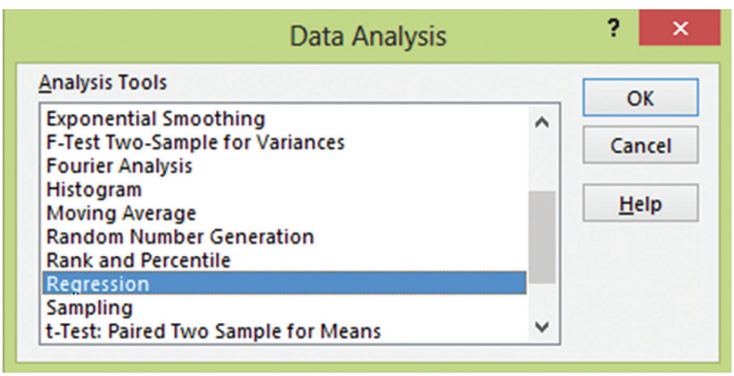
## Regression Analysis for Estimation of Length Weight Parameters

We can use  Microsoft Excel to do the analysis using the regression analysis tool.

Select Data from the Main Menu and  Select Data Analysis

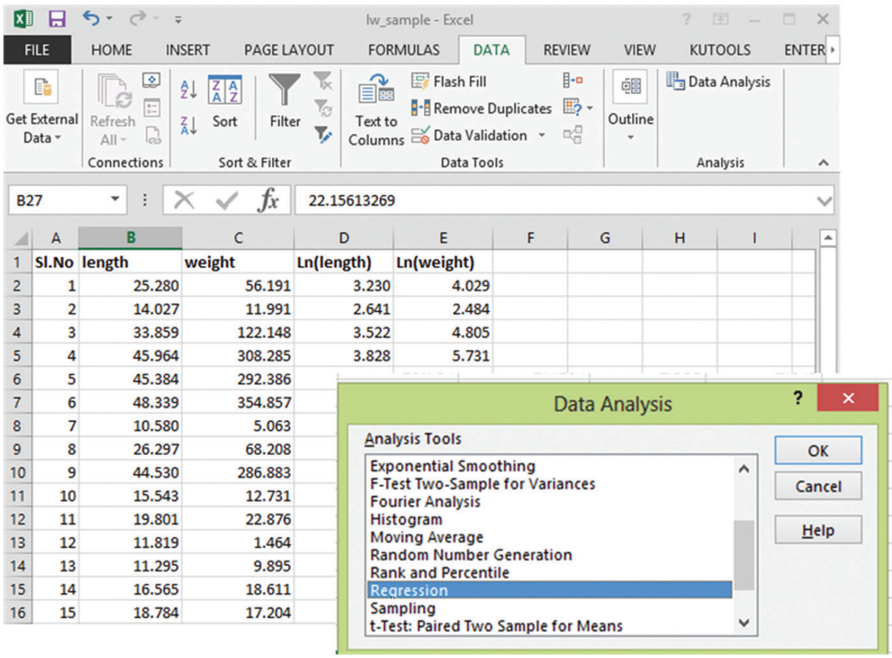



Select 'Regression' from the 'Data Analysis' dialog box and click OK.



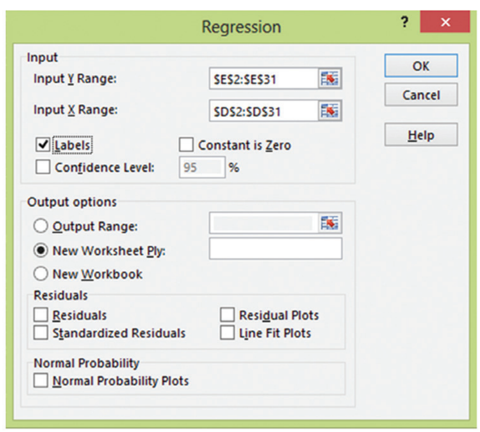
The following example demonstrates the use of this tool for estimation of length weight parameters.

Enter the data on length and weight of samples in two columns as shown in below. Generate two columns as the logarithmic values of the length and weight by using the natural logarithm function 'ln'. The transformed data will be used for estimation of parameters. To run the regression routine select **Data** from the main menu, and select **Data Analysis**. Again select Regression from the dropdown menu.

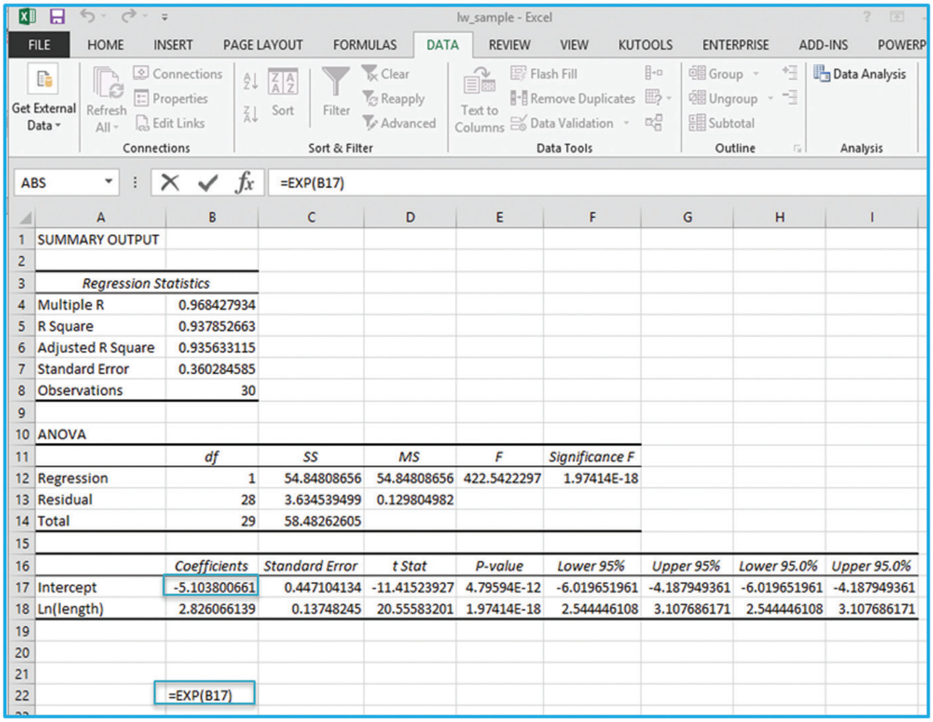



You will be presented with the following dialog box:

Specify the cells containing log transformed weight data and label for "Input Y Range:" (E21:E31). For "Input X Range:" specify the cells containing log transformed length data and label (D21:D31). Check the "Labels" box (since you included data labels in your input ranges), select the New Worksheet Ply under "Output options" and click OK.



The output will be obtained in a new sheet as given below.



| | A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|---|
| 1 | SUMMARY OUTPUT | | | | | | | | |
| 2 | | | | | | | | | |
| 3 | *Regression Statistics* | | | | | | | | |
| 4 | Multiple R | 0.968427934 | | | | | | | |
| 5 | R Square | 0.937852663 | | | | | | | |
| 6 | Adjusted R Square | 0.935633115 | | | | | | | |
| 7 | Standard Error | 0.360284585 | | | | | | | |
| 8 | Observations | 30 | | | | | | | |
| 9 | | | | | | | | | |
| 10 | ANOVA | | | | | | | | |
| 11 | | *df* | *SS* | *MS* | *F* | *Significance F* | | | |
| 12 | Regression | 1 | 54.84808656 | 54.84808656 | 422.5422297 | 1.97414E-18 | | | |
| 13 | Residual | 28 | 3.634539499 | 0.129804982 | | | | | |
| 14 | Total | 29 | 58.48262605 | | | | | | |
| 15 | | | | | | | | | |
| 16 | | *Coefficients* | *Standard Error* | *t Stat* | *P-value* | *Lower 95%* | *Upper 95%* | *Lower 95.0%* | *Upper 95.0%* |
| 17 | Intercept | -5.103800661 | 0.447104134 | -11.41523927 | 4.79594E-12 | -6.019651961 | -4.187949361 | -6.019651961 | -4.187949361 |
| 18 | Ln(length) | 2.826066139 | 0.13748245 | 20.55583201 | 1.97414E-18 | 2.544446108 | 3.107686171 | 2.544446108 | 3.107686171 |
| 19 | | | | | | | | | |
| 20 | | | | | | | | | |
| 21 | | | | | | | | | |
| 22 | | =EXP(B17) | | | | | | | |

The output will give regression statistics, ANOVA and the estimates of coefficients. The estimate of parameter 'a' is calculated from the value given against intercept and the estimate of parameter 'b' is that given against Ln(length) coefficient (here it is the value against 'ln(Length)' which is 2.826). The estimate of 'a' is calculated as the exponent of the intercept value which can be obtained by using the '**exp'** function. For example here the intercept value is in cell B17 and to obtain the estimate of 'a' in a blank cell use the function '=exp(B17)' and we get the value of a as 0.00607.

The goodness of fit of the regression model is indicated by the 'R square' value in the output. It should be high for the relationship fitted to be good. In the example it is 0.96 indicating a good fit. The maximum value of 'R square' is 1.0 and the minimum is zero.

Using the estimated values of the parameters and the original data we can calculate the expected values of weight for the lengths in the sample data. This is done by substituting the estimated values in the relationship $W = a\,L^b$ and calculating the weights corresponding to each length in the sample.

## Statistical Test for b=3 (Isometric Relationship)

In statistical test of hypothesis this is testing for the null hypothesis $H_0$ : b =3 against the alternative hypothesis $H_1$ : b≠3. The test criterion for this statistical test is a Student's t statistic with (n-2) degrees of freedom where n is the total number of observations.

Since this test criterion is for a linear regression, for the length-weight relationship situation we should use the log transformed values for the X and Y variables. Therefore, X values are the log transformed values of length and Y values are the log transformed values of the weights.

The test statistics for this is

$$t_{n-2} = \frac{(b-3)\sqrt{(n-2)\sum_{i=1}^{n}(x_i-\bar{x})^2}}{\sqrt{\sum_{i=1}^{n}(y_i-\bar{y})^2 - b^2\sum_{i=1}^{n}(x_i-\bar{x})^2}}$$

This value has to be compared with the table value of t for n-2 d.f for making inferences about the null hypothesis.

If the value of Student's t is higher than the calculated value, we accept the null hypothesis that b=3. In that case we infer that the length weight relationship is said to be isometric or there is cubic relationship between length and weight.

The length-weight relationship in fishes can be affected by a number of factors including season, habitat, gonad maturity, sex, diet, and stomach fullness, health and preservation techniques, and differences in the length ranges of the specimen caught. The exact relationship between length and weight differs among species of fish according to their inherited body shape, and within a species according to the condition (robustness) of individual fish. Condition sometimes reflects food availability and growth within the weeks prior to sampling. But, condition is variable and dynamic. Individual fish within the same sample vary considerably, and the average condition of each population varies seasonally and yearly.