ESTIMATION OF MORTALITY

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Fish as a natural resource follows most of the established behavior expected of ay similar animal. The birth and ensuing recruitment, growth, reproduction and death, technically referred to as mortality are well defined phases of any animal's life time and fishes are no exceptions to this. As is evident from its logic, fish populations increase in their abundance, popularly termed as biomass, by birth of animals or by growth apart from occasional immigrations. The loss of animals is mostly through death (mortality) which could occur due to ageing, natural mortality, or due to fishing, fishing mortality apart from the predation inflicted by larger animals in the sea. Hence mortality phenomenon happens to be the single most important cause of change in abundance of fish in any defined population or technically referred to as stock. By its sheer importance as the leveling force in face of animals with varying degrees of reproduction, mortality assumes an important position in the study of dynamics or fluctuations in the biomass of a given resource. Like its growth counterparts mortality too has well laid conceptualization coupled with clearly defined procedures of measurement or technically termed as estimation. Thus the phenomenon of rate of loss of animals in a particular population is a parameter to be estimated with the sampled animals. The measurements taken from the sampled individuals help an assessor to find out the composition of fish available at various ages and such information collected over a period of time will enable the observer to find out the rates at which fish of a particular age die due to natural and unnatural causes.

The inevitability of the mortality phenomenon can be understood by the fact that for a group of contemporarily hatched fish the number can only dwindle over time. The contemporaries or those individuals who were hatched almost at the same epoch are technically termed as cohorts. The phenomenon of mortality applies to each such group of cohorts and how they decline in number through time. To clearly delineate this process of decline in numbers it is essential to follow the fate of the cohort. As mentioned earlier cohort is a batch of fish of all of approximately the same age and belonging to the same stock. (Sparre and Venema 1998). All fish of a cohort are assumed to have the same age at given time so that they all attain the recruitment age at the same time. In the context of mortality one is interested in the number of survivors from a cohort as a function of age. As mortality is split into natural and fishing induced ones, estimating the mortality entails the determination of total mortality (natural

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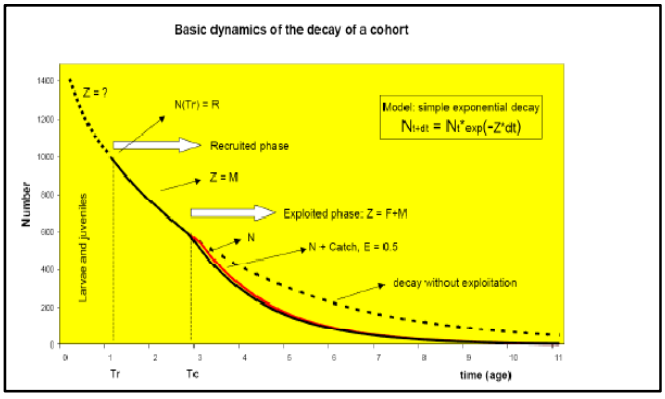
mortality + fishing mortality) first and then splitting this into natural and fishing mortalities as appropriate.

The progress of a cohort over time is displayed below in figure. In a cohort model it is assumed that R individuals are recruited into the fishery at the age t_r (denoting age at recruitment). From this age fish are exposed to some degree of natural mortality ,M. After certain time these fish are exposed to fishing at age t_c (age at first capture) denoted by F for fishing mortality. At some point tmax the older fish are not vulnerable to fishing. This setup assumes an all or none type of selection popularly referred to as knife edge selection, whereby at tc either none or all fish in an age class are either recruited or not or vulnerable or not, and once vulnerable all age classes are vulnerable. (Sparre and Venema 1992)



Dynamics of a Cohort

The dynamics of similarly aged fish of a stock are assumed to follow the model of natural decay, whereby the reduction in numbers due to total mortality is an exponential function of the number of cohorts at the beginning of the period. Notationally the rate of change in numbers or number of losses or number of animals died in a small epoch is given by the following equation

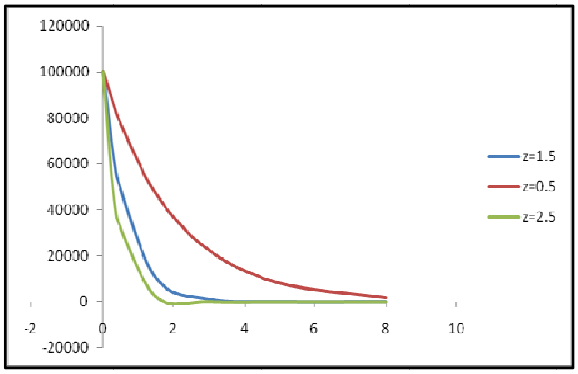
$$\frac{\Delta N(t)}{\Delta t} = -Z * N(t)$$

where the deltas indicate the change in numbers and a small interval of time, say one day or week etc. Z is the coefficient of reduction or popularly known as rate of annual instantaneous mortality usually scaled to account for one year. N(t) indicates the number of individuals alive at time t, preferably converted to years. This total mortality is supposed to be the arithmetic sum of natural mortality M and fishing mortality F. Notational depiction is as follows.

$$Z=M + F$$

A gentle mathematical juggling would yield the number of individuals alive at time t which follows the time of recruitment of the cohorts into the fishery at Tr could lead to an equation

N(t)=N(Tr)*exp(-Z(t-Tr)). That is the number of individuals available at the present time in years is a function of the difference between the time at recruitment and the present



time and also the number of live individuals at the time of recruitment. Of course this whole relationship assumes that the rate of instantaneous mortality Z remains constant throughout. As a corollary it can be said that higher the value of Z the faster would be the decline in numbers and the lower would be the maximum age.

As an example let us look at a case where the recruitment number N(Tr) of a cohort is 1,00,000 and total instantaneous mortality Z = 1.5 per year. Assuming Δt as one day that is 1/365thof a year then, the number of survivors at different time intervals is given in the table.

Age of cohorts (t years)	Time interval in years	Number of survivors (N(Tr)*exp(-Z*(t-Tr)))
tr	0	100000
tr+ one day	0.00274	99590
tr+0.2	0.2	74082
tr+0.3	0.3	63763
tr+0.4	0.4	54881
tr+0.4+one day	0.40274	54656
tr+1.5	1.5	10540
tr+3.0	3	1111
tr+5.0	5	55
tr+8.0	8	1

As it can be seen that the loss is very severe in the initial phases as compared to the last stage like 8 years whereby the cohorts effectively vanish. The steepness depends upon the value of Z and it is better depicted in the following chart.

As is evident the decline is the steepest in the z=2.5 case and it is the slowest in z=0.5 case. The X axis indicates the time gap after Tr in years and the Y axis entries indicate the number of surviving animals.

After the animals obtain age of first capture they are most vulnerable to fishing mortality, whereas upto the age of recruitment the decline in numbers is mostly due to predation or disease ie natural mortality. Assuming that Z=F+M, the number of cohorts caught in a period from t1 to t2 in years is expressed as the function of total and fishing mortality as follows.

$$C(t1,t2)=F/Z*[(N(t1)-N(t2)]$$

Wherein N(t1) and N(t2) are the animals available at time periods t1 and t2. This equation is of extreme importance in fish stock assessment and is famously referred as Baranov's equation or Catch Equation. The fraction F/Z is also very important from

assessment point of view and is popularly referred to as "Exploitation Rate". At the same period the number of animals dying due to natural causes is

D(t1,t2)= M/Z*[N(t1)-N(t2)]

The catch equation can be rewritten by involving the number of individuals at the beginning ie t1 as follows:

C(t1,t2)= N(t1)*F/Z*(1-exp(-Z(t2-t1)))

One major assumption which is the soul of this entire conceptualization is the fact that during the time interval (t1,t2) the situation at the ground is not fluctuating enough to influence the mortality rates, F and M. But criticisms are always possible on the count that natural mortality rates tend to differ with aging and younger fishes which are possibly smaller in size are less prone to fishing mortality as compared to their older counterparts.

Another conceptualization based on catch equation is the "Average number of survivors during the time period (t1,t2)" which is given by

$$\bar{N}(t1,t2) = N(t1) * \frac{1 - e^{-Z*(t2-t1))}}{Z * (t2 - t1)}$$

Estimation of Total Instantaneous Mortality (Z)

a) From Catch Rates

There are very many ways of estimating Z from the data collected from research fishery. One such method is the method based on catch rates or Catch Per Unit Effort (CPUE) which is the ratio of total quantity of fish caught to the total number of units of gear utilized to catch the same. When the fish are caught with the same gear whose catchability coefficient (q) with respect to a particular resource is constant, the proportion of surviving members of the cohorts at two time periods (t1,t2) is equal to the ratio of the catch rates at the two time periods recorded by exploratory survey. That is

$$\frac{N(t2)}{N(t1)} = \frac{CPUE(t1)}{CPUE(t2)}$$

A slight modification of the catch equation would lead to the following relationship when the number of cohorts available at the time limits vizN(t1) and N(t2) are known.

$$Z = \frac{1}{t2 - t1} * \log\left(\frac{N(t1)}{N(t2)}\right)$$

Using the previous two relationships it can be derived

$$Z = \frac{1}{t2 - t1} * \log\left(\frac{CPUE(t1)}{CPUE(t2)}\right)$$

When the data available pertains to commercial fisheries, where the time series is on quarterly or annual basis, the equation used could be similar to the one described previously and the CPUE is calculated as the catch of cohort during the period (t1,t2) divided by the effort during that period. The catch rate can then be expressed as the product of average number of survivors in the period (t1,t2) and the catchability coefficient of the gear.

b) Heincke's method

Assuming that mortality rate (Z) is constant throughout the life of an individual, the following equation holds based on certain algebraic norms.

$$Z = -\ln\ (\sum_{t=1}^{\infty} N(t) / \sum_{t=0}^{\infty} N(t)$$

which is called the Heincke's equation. In plain words the mortality rate is the negative value of the ratio between the number of surviving individuals from age 1 to those surviving from age 0. Substituting CPUE's at each year in the place of N(t)'s this equation assuming that they are proportional the same reads as

$$Z = -ln \frac{CPUE(1) + CPUE(2) + CPUE(3\ and\ above)}{CPUE(0) + CPUE(1) + CPUE(2) + CPUE(3\ and\ above)}$$

c) Robson- Chapman Method

Another estimate of Z is proposed by Robson and Chapman (Sparre&Venema, 1992) and the formula is

$$Z = -ln \frac{N(1) + 2 * N(2) + 3 * N(3) + \cdots}{N(0) + 2N(2) + 3N(2) + 4(N3) + \cdots - 1}$$

d) Linearised Catch Curve Method

Ideally for estimating most of the parameters including the mortality rate, the type of data required is the number of sampled and raised animals belonging to a cohort at various age categories. However in fishery sampling age determination is a time and manpower consuming exercise and invariably aging is done by using the length of the animals sampled and their categories thereof. Here length is used as an alibi for age. Further it is worth recalling that age and length are functionally linked through the Von Bertalanffy Growth Function (VBGF). Using the inversion of the VBGF length can be converted into age. The specific relationship is as follows:

$$t(L)Z = t_0 - \frac{1}{K} * \ln\ (1 - \frac{L}{L_\infty})$$

where t(L) is the age at length L units (cm or mm) and t_0, L_∞ and K are the classical VBGF parameters. Using this in the equation relating the logarithm of catch rate over a small time interval and the mid –time interval which is as follows:

$$\ln\left(\frac{C(t,t+\Delta t)}{\Delta t}\right) = c - Z * (t + \frac{\Delta t}{2})$$

which in turn can be rewritten using the catch and length information as

$$\ln\frac{C(L_1, L_2)}{\Delta t} = c - Z * t\left(\frac{L_1 + L_2}{2}\right).$$

Here the change in time

$$\Delta t \text{ isgiven by } \frac{1}{K} * \ln\left(\frac{L_\infty - L_1}{L_\infty - L_2}\right)$$

Thus from this linear function, the total instantaneous rate of mortality can be estimated as the negative slope. It can be noted that in the previous equations c is a term which is made of constant terms or in other words by terms which are not involving either time or length at different classes.

To put this linearised catch curve method into action, a plot of

$$\ln\frac{C(L_1,L_2)}{\Delta t} \text{against} t\left(\frac{L_1+L_2}{2}\right)$$

has to be made. Only the stable range of t values which are in the fully exploited range of the animal's life and which also is not close to t_∞ (age at maximum length of the animal) must be included for the computation of the coefficients of regression. This procedure is partially subjective which must be given due care.

Example

A worked out example of estimating total instantaneous mortality rate Z from length frequency data is given below.

The case is that of Upeneusvittatus from Manila Bay, Philippines (quoted in Sparre and Venema 1992) and the length intervals and catch numbers of the pseudo cohorts is given below. The VBGF parameters are K=0.59 per year; L_∞=23.1 cm; and t_0=0

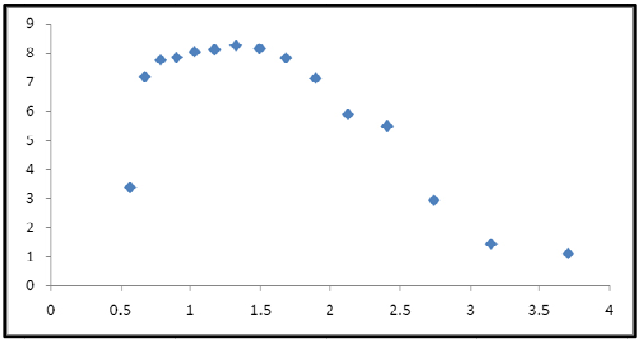
L1-L2	C(L1,L2)	t(L1)	Δt	t(L1+L2)/2	ln(C(L1,L2)/Δt)
6-7	3	0.51	0.102	0.56	3.381395
7-8	143	0.612	0.109	0.665	7.179252
8-9	271	0.721	0.116	0.778	7.756284
9-10	318	0.837	0.125	0.898	7.841493
10-11	416	0.961	0.135	1.027	8.033166
11-12	488	1.096	0.146	1.168	8.114464
12-13	614	1.242	0.16	1.32	8.252576
13-14	613	1.402	0.177	1.488	8.14997
14-15	493	1.579	0.197	1.675	7.825061
15-16	278	1.776	0.223	1.884	7.128205
16-17	93	2	0.257	2.123	5.891279

17-18	73	2.257	0.303	2.402	5.484482
18-19	7	2.56	0.37	2.735	2.940162
19-20	2	2.93	0.474	3.151	1.439695
20-21	2	3.404	0.66	3.702	1.108663
21-22	0	4.064	1.096	4.525	-
22-23	1	5.16	4.064	6.188	-1.40217
23-24	1	-	-	-	-

To select the most appropriate portion of the length intervals a plot of

$\ln \frac{C(L_1,L_2)}{\Delta t}$against$t\left(\frac{L_1+L_2}{2}\right)$

is made and in the above case it looks like this:



As is evident from the scatter the first 7 observations and the last two observations do not follow the steady fall pattern and hence can be avoided. So only the points in the mean time range 1.5 to 3.15 are considered for estimating the regression coefficients. In this present case the estimated slope is -4.19433 and hence the estimated Z rate is 4.19.

e) The cumulated catch curve method

Another approach to estimate Z from length frequency data is the Cumulated Catch Curve method propounded by Jones and Van Zalange. The main difference here is in the time range (t1,t2) t2 is assumed to be very large to be near ∞ and that would lead the linearised catch curve equation to become

LnC(t, ∞)=d- Z*t where C(t, ∞) is called cumulated catch curve equation. Then the Jones and Van Zalinge equation for length converted catch curve would be

$$\ln\left(C(L,L_\infty)\right) = a + \frac{Z}{K}\ln\left(L_\infty - L\right)$$

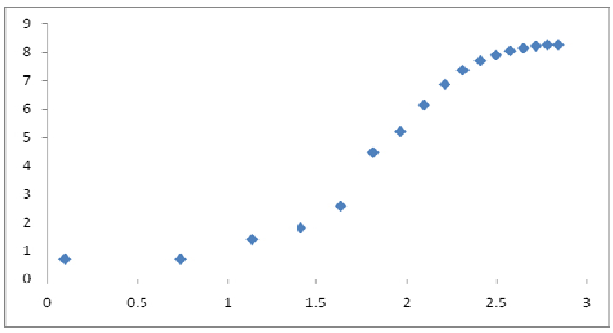
After selecting the appropriate portion of the scatter between

$$\ln\left(C(L,L_\infty)\right)\text{and } \ln\left(L_\infty - L\right).$$

Then from the slope the Z is estimated as slope * K. For the previous example the cumulated catch curve approach is done as follows:

L1-L2	C(L1,L2)	C(L1,L∞)	ln(C(L1,L∞))	ln(L∞-L1)
6-7	3	3816	8.246958	2.839078
7-8	143	3813	8.246172	2.778819
8-9	271	3670	8.207947	2.714695
9-10	318	3399	8.131237	2.646175
10-11	416	3081	8.033009	2.572612
11-12	488	2665	7.887959	2.493205
12-13	614	2177	7.685703	2.406945
13-14	613	1563	7.354362	2.312535
14-15	493	950	6.856462	2.208274
15-16	278	457	6.124683	2.091864
16-17	93	179	5.187386	1.960095
17-18	73	86	4.454347	1.808289
18-19	7	13	2.564949	1.629241
19-20	2	6	1.791759	1.410987
20-21	2	4	1.386294	1.131402
21-22	0	2	0.693147	0.741937
22-23	1	2	0.693147	0.09531
23-24	1	1	0	-2.30259

The plot based on the last two columns is shown here. From the plot it can be seen that the most appropriate range to be used for estimating the slope is the x range from 1.41 to 2.31 and the corresponding slope value is 6.51. Hence the estimated Z rate = Slope*K= 6.51* 0.59= 3.84.



f) Beverton and Holt's Z-equation based on length data

Beverton and Holt (Sparre and Venema 1992) have shown that there exists a functional relationship between Z and the average length of fish $\bar{L}$ which is given by

$$Z = K * \frac{L_\infty - \bar{L}}{\bar{L} - L'}$$

where L′ is some length for which all fish of that length and longer are under full exploitation and it is the lower limit of the class interval of lengths from which point full exploitation is presumed.

For example if the VBGF parameters of a cohort are K=0.45 per year and L∞ = 100 cm and if it is assumed that L′=45 cm then the Z estimates for the following data are as given below:

Length Group	Mid Length	N(1960)	N(1970)	N(1980)	N(1960)*Mid Length	N(1970)*Mid Length	N(1980)*Mid Length
45-50	47.5	256	268	212	12160	12730	10070
50-55	52.5	237	226	161	12442.5	11865	8452.5
55-60	57.5	211	180	116	12132.5	10350	6670
60-65	62.5	187	141	79	11687.5	8812.5	4937.5
65-70	67.5	161	105	52	10867.5	7087.5	3510
70-75	72.5	138	76	31	10005	5510	2247.5
75-80	77.5	113	50	17	8757.5	3875	1317.5
80-85	82.5	87	30	8	7177.5	2475	660
85-90	87.5	62	15	3	5425	1312.5	262.5
90-95	92.5	36	6	1	3330	555	92.5
95-100	97.5	12	1	0	1170	97.5	0
	Sum	1500	1098	680	95155	64670	38220
	$\bar{L}$				63.43667	58.898	56.20588

Where N(1960) indicates numbers caught in year 1960 and so on. The mean length here is a weighted average of the lengths detailed.

Based on these figures the Z values for various years are as follows:

$$Z(1960)=0.3 * \frac{100-63.44}{63.44-45} = 0.6 \text{ per year}$$

$$Z(1970)=0.3 * \frac{100-58.90}{58.90-45} = 0.9 \text{ per year}$$

$$Z(1980)=0.3 * \frac{100-56.21}{56.21-45} = 1.2 \text{ per year}$$

g) Power- Wetherall method

As a special application of the Beverton- Holt's Z- equation it can be expressed that

$\bar{L}$=a + b*L′ where Z/K=-(1+b)/b and L_∞=-a/b or alternatively

b=-K/(Z+K) and a=-b* L_∞

This means that plotting $\bar{L}$ -L′ against L′ gives the estimates of a and b and from them the parameters L_∞ and Z can be estimated.

h) Pauly's empirical equation for Natural Mortality Estimation

Pauly (Sparre and Venema 1992) made regression analysis to functionally link natural mortality M with VBGF parameters and climatic parameters and the empirical formula arrived by him is given below:

Rate of Natural Mortality per Year (M)= $-0.0152-0.279*\ln L_\infty+0.6543*\ln K+0.463*\ln T$
where T is the average annual temperature at the surface in degrees centigrade.

The following table gives the estimates of natural mortality for various combinations of T and VBGF parameters.

T=5°C					T=25°C			
L∞	K=0.1	0.5	1.0	2.0	0.1	0.5	1.0	2.0
10	0.24	0.7	1.1	1.7	0.51	1.5	2.3	3.6
80	0.14	0.38	0.6	1.0	0.29	0.8	1.3	2.0
200	0.10	0.30	0.47	0.7	0.22	0.6	1.0	1.6

Method of Computing

The above discussed methods of estimating rates of mortalities can be implemented practically either by manual means (highly exhausting) or by using computer based spread sheets or by software custom made for this purpose.

Suggested Reading

Sparre, P and Venema, S.C (1992) Introduction to Tropical Fish Stock Assessment. Part 1.Manual, FAO Fisheries Technical Paper No. 306.1, Rev. 1., Rome.376 p.